

TRIGONOMETRY

Thursday, June 21, 1962 — 9:15 a.m. to 12:15 p.m., only

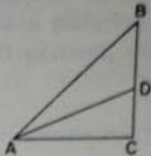
Name of pupil.....Name of school.....

Name of teacher.....

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form.

- 1 Express $\sin (-160^\circ)$ as a function of a positive acute angle. 1.....
- 2 In circle O whose radius is 4 inches, central angle AOB intercepts a minor arc of 8 inches. Find the number of radians in angle AOB . 2.....
- 3 Find the smallest positive acute angle for which $2 \sin^2 x - 1 = 0$. 3.....
- 4 Find the numerical value of $2 \sin^2 2x$ if $x = \frac{\pi}{6}$. 4.....
- 5 What is the numerical value of $\cos 75^\circ 12'$? 5.....
- 6 If $\log \tan A = 0.0537$, find to the nearest minute the positive acute angle A . 6.....
- 7 Find the numerical value of $\cos \pi + \cos 2\pi + \cos 3\pi$. 7.....
- 8 In triangle ABC , $a = 8$, $b = 5$, $\sin C = 0.90$ and $\cos C = 0.44$. Find to the nearest integer the area of triangle ABC . 8.....
- 9 Given triangle ABC . Express b in terms of a , A and B . 9.....
- 10 In triangle ABC , $a = 4$, $b = 5$ and $c = 6$. Find the value of $\cos C$. 10.....
- 11 If the values of x in the equation $\tan^2 x = 4$ are restricted to positive angles less than 360° , the set of solutions for the equation has four members. Three of these members, to the nearest degree, are 63° , 117° and 243° . Find to the nearest degree the remaining member of this solution set. 11.....
- 12 Find the positive value of $\sin (\arctan \sqrt{2})$. 12.....

- 13 If $\cos x = -0.28$, find in simplest form the positive value of $\cos \frac{1}{2}x$. 13.....
- 14 Express as an inverse function the value of x for which $2\sqrt{1 - \cos x} - 1 = 0$. 14.....
- 15 In the diagram shown, ABC is an isosceles right triangle, angle $C = 90^\circ$, $AC = 10$ and AD bisects angle BAC . Find to the nearest tenth the length of CD . 15.....
- 
- 16 Find the smallest positive value of x satisfying these two conditions: $\tan x = -1$ and $\cos x$ is negative. 16.....

Directions (17–27): Write on the line at the right of each of the following the number preceding the expression that best completes the statement.

- 17 The number of degrees in an angle of 0.5 radian is
- (1) 90° (3) $\frac{90^\circ}{\pi}$
- (2) 180° (4) $\frac{180^\circ}{\pi}$ 17.....
- 18 The expression $\cos(2\pi + x) + \sin(\pi + x)$ is equal to
- (1) $\cos x + \sin x$ (3) $-\cos x - \sin x$
- (2) $-\cos x + \sin x$ (4) $\cos x - \sin x$ 18.....
- 19 The expression $\sec x - \tan x \sin x$ is equivalent to
- (1) $1 + \sin x$ (3) $\cos x$
- (2) $1 - \sin x$ (4) $-\cos x$ 19.....
- 20 The expression $2 \cos 3x \cos x$ is equivalent to
- (1) $\sin 4x + \sin 2x$ (3) $\sin 2x + \sin x$
- (2) $\cos 4x + \cos 2x$ (4) $\cos 2x + \cos x$ 20.....
- 21 The expression $\sin(x - 30^\circ)$ equals
- (1) $\frac{\sqrt{3} \sin x - \cos x}{2}$ (3) $\frac{\cos x - \sqrt{3} \sin x}{2}$
- (2) $\frac{\sqrt{3} \cos x - \sin x}{2}$ (4) $\frac{\sin x - \sqrt{3} \cos x}{2}$ 21.....
- 22 If $\tan \theta = \sqrt{2}$, $\tan 2\theta$ equals
- (1) $2\sqrt{2}$ (3) $\frac{2\sqrt{2}}{1 - \sqrt{2}}$
- (2) $-2\sqrt{2}$ (4) $\frac{2\sqrt{2}}{1 + \sqrt{2}}$ 22.....
- [2]

23 If $\cos 2x$ is represented by k , then $\sin^2 x$, when represented in terms of k , is

(1) $\frac{1-k}{2}$

(3) $\frac{1+k}{2}$

(2) $1-k^2$

(4) $1-2k^2$

23.....

24 If $S = \frac{10}{\tan A}$, then a correct expression for $\log S$ is

(1) $1 - \tan A$

(3) $\log \cot A$

(2) $10 - \log \tan A$

(4) $1 + \log \cot A$

24.....

25 As x varies from 0 to 2π radians, the expression $2 \sin \frac{x}{2}$ reaches a maximum value once. The value of x that results in this maximum value is

(1) 1

(3) π

(2) 2

(4) 2π

25.....

26 If $\cos 3x = \sin 60^\circ$, then x equals

(1) 10°

(3) 40°

(2) 30°

(4) 70°

26.....

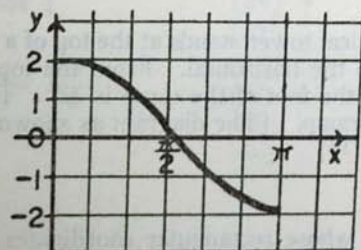
27 A portion of the graph of an equation is shown at the right. The equation is

(1) $y = 2 \cos \frac{1}{2}x$

(2) $y = 2 \cos x$

(3) $y = 2 \cos 2x$

(4) $y = \cos 2x$



27.....

Directions (28–29): For values of x for which the following statements are defined, indicate whether *each* of the statements is true for

(1) all real values of x ,

(2) some but not all real values of x ,

(3) no real values of x ,

by writing on the line at the right the number 1, 2 or 3.

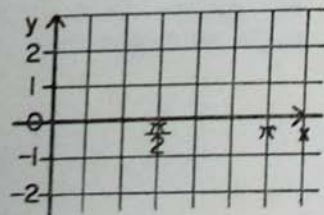
28 $\sin 2x = 2 \sin x$

28.....

29 $\cos^2 \frac{x}{2} + \sin^2 \frac{x}{2} = \frac{1}{2}$

29.....

30 On the coordinate axes at the right, *sketch* the graph of $y = \sin 2x$ from $x = 0$ to $x = \pi$.



Part II

Answer four questions from this part. Show all work unless otherwise directed.

- 31 Find all positive values of x less than 360° which satisfy the equation $6 \sin^2 x - \cos x - 4 = 0$.
[Express approximate values of x to the nearest degree.] [10]

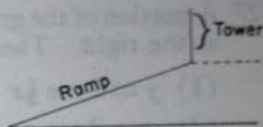
- 32 a Starting with the formula for $\cos(x + y)$, derive a formula for $\cos 2x$ in terms of $\cos x$. [5]
b Starting with the formula for $\cos 2x$, derive a formula for $\cos \frac{1}{2}u$ in terms of $\cos u$. [5]

- 33 A metal frame is constructed in the form of an isosceles trapezoid. Each base angle measures $73^\circ 30'$, the shorter base is 8.0 feet and each of the nonparallel sides is 5.0 feet. Find to the nearest tenth of a foot the length of a diagonal brace of this frame. [4, 6]

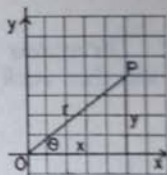
- 34 a Show that the expression $\frac{\sin 2x}{\sin x} - \frac{\cos 2x}{\cos x}$ is equivalent to $\sec x$. [6]

- b Starting with the formula for $\tan 2x$, derive the formula $\cot 2x = \frac{\cot^2 x - 1}{2 \cot x}$. [4]

- 35 A 42-foot vertical tower stands at the top of a ramp which has an inclination of 18° to the horizontal. From the top of the tower the angle of depression of the foot of the ramp is 32° . Find to the nearest foot the length of the ramp. [The diagram as shown represents a vertical cross section.] [10]



- 36 P is a point whose rectangular coordinates are represented by x and y , as shown. The distance from the origin O to point P is represented by r , and the angle that OP makes with the positive portion of the x -axis is represented by θ .



- a Show that the equation $r = 4 \cos \theta$ may be converted to the equation $x^2 - 4x + y^2 = 0$. [5]

- b Show that the equation $x^2 + (y - 2)^2 = 4$ may be converted to the equation $r^2 = 4r \sin \theta$. [5]

FOR TEACHERS ONLY

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INSTRUCTIONS FOR RATING TRIGONOMETRY

Thursday, June 21, 1962 — 9:15 a.m. to 12:15 p.m., only

Use only red ink or pencil in rating Regents papers. Do not attempt to correct the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 17–27, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3 or 4.

- | | | |
|--|--------------------------------------|--------|
| (1) $-\sin 20^\circ$ or $-\cos 70^\circ$ | (13) 0.6 | (23) 1 |
| (2) 2 | (14) $\arccos \frac{3}{4}$ | (24) 4 |
| (3) 45° or $\frac{\pi}{4}$ | (15) 4.1 | (25) 3 |
| (4) $\frac{3}{2}$ | (16) 135° or $\frac{3\pi}{4}$ | (26) 1 |
| (5) 0.2554 | (17) 3 | (27) 2 |
| (6) $48^\circ 32'$ | (18) 4 | (28) 2 |
| (7) -1 | (19) 3 | (29) 3 |
| (8) 18 | (20) 2 | |
| (9) $\frac{a \sin B}{\sin A}$ | (21) 1 | |
| (10) $\frac{1}{8}$ | (22) 2 | |
| (11) 297 | | |
| (12) $\sqrt{\frac{2}{3}}$ | | |

Part II

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics*. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

- | | |
|-----------------|-------------------|
| (31) 60° | (33) Analysis [4] |
| 132° | 10.6 [6] |
| 228° | |
| 300° [10] | (35) 147 [10] |

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