TRIGONOMETRY

Thursday, June 22, 1961 — 9:15 a.m. to 12:15 p.m., only

Name of pupil..Name of school..................................

Name of teacher..

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form.

1 Express $\sin(-160°)$ as a function of a positive acute angle.　　1................

2 If A is a positive acute angle, express $\tan A$ in terms of $\cos A$.　　2................

3 Angle A terminates in quadrant IV and $\cos A = \dfrac{5}{13}$. Express

$\sin 2A$ as a fraction.　　3................

4 Find the number of radians in $135°$.　　4................

5 Angle A is a positive acute angle. Express $\cot A \cos A$ in terms of $\sin A$.　　5................

6 Give in radians the period of $y = 2 \sin x$.　　6................

7 What is the minimum value of $\sin \dfrac{x}{2}$?　　7................

8 Find the number of degrees in the positive acute angle θ for which $2 \sin \theta \cos \theta - \cos \theta = 0$.　　8................

9 Express $\tan 4A$ in terms of $\tan 2A$.　　9................

10 What is the smallest positive value of A if $A = \arccos(-1)$?　　10................

11 In triangle ABC, $a = 1.8$, $\sin A = 0.9$ and $b = 1$. Find the value of $\sin B$.　　11................

12 In triangle ABC, $a = \sqrt{19}$, $b = 3$ and $c = 2$. Find the number of degrees in angle A.　　12................

13 In triangle ABC, $C = 120°$, $a = 10$ and $b = 7$. Find the value of $\tan \dfrac{A-B}{2}$.　　13................

[1]　　[OVER]

14 In triangle ABC, $a = 2$, $b = 3$ and $C = 60°$. What is the area of triangle ABC?

14.....................

15 If angle A terminates in quadrant III and tan $A = \frac{4}{3}$, express in radical form the positive value of sin $\dfrac{A}{2}$.

15.....................

16 Find log tan $38° 27'$.

16.....................

17 Find to the *nearest minute* the positive acute angle whose cosine is 0.2885.

17.....................

18 How many distinct noncongruent triangles, if any, can be constructed using the data $A = 30°$, $b = 6$ and $a = 5$?

18.....................

19 Two forces acting upon a body make an angle of $60°$ with each other. If the magnitudes of the forces are 2 pounds and 4 pounds, respectively, find the number of pounds in the magnitude of the resultant.

19.....................

20 If $\csc^2 \theta = 4$, find the values of sin θ.

20.....................

Directions (21–26): Write on the line at the right of *each* of the following the *number* preceding the expression that best completes the statement.

21 A course S $60°$ W is the same, when measured clockwise from the north, as a course
(1) 060   (3) 240
(2) 120   (4) 300

21......

22 The expression sin $40°$ + sin $20°$ is equal to
(1) sin $(40° + 20°)$ (3) 2 cos $30°$ cos $10°$
(2) 2 sin $60°$ cos $20°$ (4) 2 sin $30°$ cos $10°$

22......

23 If log sin $\theta = a$, then log 100 sin^2 θ equals
(1) $2 + 2a$ (3) $100 + 2a$
(2) $2a^2$ (4) $4a$

23......

24 In a circle of radius r, a chord of length r intercepts a minor arc whose central angle is
(1) less than 1 radian
(2) equal to 1 radian
(3) greater than 1 radian but less than 2 radians
(4) equal to 2 radians

24......

25 As x increases from $180°$ to $360°$, cos x
(1) increases, then decreases
(2) decreases, then increases
(3) decreases throughout the interval
(4) increases throughout the interval

25......

26 For values of x greater than or equal to $0°$ but less than $360°$, the number of solutions of the equation sin x $(2 \sin x - 1) = 0$ is
(1) 1   (3) 3
(2) 2   (4) 4

26......

Directions (27–30): Indicate whether *each* of the following statements is true for
  (1) all real values of *A*,
  (2) some but not all real values of *A*,
  (3) no real values of *A*,

by writing on the line at the right the number 1, 2 or 3.

27 $2 \cos^2 A = \cos 2A + 1$ 27......

28 $\cos (90° - A) = \cos A$ 28......

29 $\sin A > \tan A$ 29......

30 $\sin A + \sin (-A) = 0$ 30......

Part II
Answer four questions from this part. Show all work unless otherwise directed.

31 Solve the following equation for all values of *A* greater than or equal to 0° but less than 360° :
   [Express *A* to the *nearest ten minutes.*]   [10]
$$9 + 5 \sin A = 12 \cos^2 A$$

32 *a* On the same set of axes, *sketch* the graphs of $y = 2 \sin x$ and $y = \cos \dfrac{x}{2}$ as *x* varies from
     0 to 2π radians. [Label each curve with its equation.]   [4, 4]

  b From the graphs made in answer to part *a*, determine the number of solutions of
     $2 \sin x = \cos \dfrac{x}{2}$ as *x* varies from 0 to 2π radians.   [2]

33 *a* *Derive* the formula for $\sin (x + y)$ if *x*, *y* and $x + y$ are positive acute angles.   [7]
  b Using the formula for $\sin (x + y)$, *derive* a formula for $\sin 2x$.   [3]
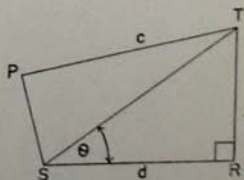
34 *a* Prove that the following equality is an identity: [7]
$$\frac{\sec x \sec y}{\tan x - \tan y} = \csc (x - y)$$

  b If $\cos 6\theta = b$ and 6θ is an obtuse angle, express $\cos 3\theta$ in terms of *b*.   [3]

35 In the accompanying diagram angle *R* is a right angle, angle
   $TSR = \theta$ and angle *TSP* is twice angle *TSR*. $PT = c$ and
   $SR = d$.

   Prove: $\sin P = \dfrac{2d \sin \theta}{c}$.   [10]

36 A captain sails his ship from point *A* on a course 040 for 2 hours at an average speed of 30
   knots (nautical miles per hour). The captain then changes course to 160 and is forced to
   reduce his average speed for this leg of the course to 20 knots. He continues on this course
   for 5 hours and arrives at point *C*. Find to the *nearest ten nautical miles* the distance *AC*. [10]

[3]

FOR TEACHERS ONLY

INSTRUCTIONS FOR RATING
TRIGONOMETRY
Thursday, June 22, 1961 — 9:15 a.m. to 12:15 p.m., only

Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 21–30, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3 or 4.

(1) — $\sin 20°$ or — $\cos 70°$

(2) $\dfrac{\sqrt{1 - \cos^2 A}}{\cos A}$

(3) — $\dfrac{120}{169}$

(4) $\dfrac{3\pi}{4}$

(5) $\dfrac{1 - \sin^2 A}{\sin A}$

(6) 2π

(7) —1

(8) 30

(9) $\dfrac{2 \tan (2A)}{1 - \tan^2 (2A)}$

(10) $180°$ or π

(11) 0.5

(12) 120

(13) $\dfrac{\sqrt{3}}{17}$

(14) $\dfrac{3\sqrt{3}}{2}$

(15) $\dfrac{3}{\sqrt{10}}$ or $\dfrac{3\sqrt{10}}{10}$

(16) $9.8998 - 10$

(17) $73°\ 14'$

(18) two

(19) $2\sqrt{7}$ or 5.3

(20) $\pm \frac{1}{2}$

(21) 3

(22) 4

(23) 1

(24) 3

(25) 4

(26) 4

(27) 1

(28) 2

(29) 2

(30) 1

[OVER]

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics.* Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

Part II

(31) 19° 30'
      160° 30'
      228° 40' [10]
      311° 20'

(32) b 3 [2]

(34) $b \sqrt{\dfrac{1+b}{2}}$ [3]

(36) 90 [10]