June 19, 1957

Part I

Answer all questions in this part. Each correct answer will receive 2½ credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form.

1. Express sin 280° as a function of a positive acute angle.   1_____

2. If $\tan x = \dfrac{4}{3}$ and x is an angle in the first quadrant,
find cos x.   2_____

3. If $\sin^2 B - \sin B = 0$, find a value of B greater than 0° and less than 180° which satisfies this equation.   3_____

4. Given triangle ABC, write an equation that could be used to solve for side a in terms of side b and functions of angle A and angle B.   4_____

5. Express sin P + sin Q as an equivalent product of trigonometric functions.   5_____

6. If $\cos x = \dfrac{5}{13}$ and x is an angle in the first quadrant, find $\cos(180° - x)$.   6_____

7. Find log tan 63° 43′.   7_____

8. Find cot 34° 36′.   8_____

9. Find the number whose logarithm is 3.5421.   9_____

10. Find the positive value of sin (arc tan 3).   10_____

11. What is the value of $\sin^2 3A + \cos^2 3A$?   11_____

12. Find the number of radians in a central angle of a circle whose radius is 5 inches if the central angle intercepts an arc 14 inches long.   12_____

13. If $\tan x = \frac{2}{3}$, find sin 2x.   13_____

14. If x is any angle, express sin $\frac{1}{2}x$ in terms of a function of x.   14_____

15. How many different triangles can be constructed, given the data $a = 7$, $b = 6$ and $A = 50°$?   15_____

16. In triangle ABC, $a = 5$, $b = 8$ and $C = 60°$. Find c.   16_____

17. In triangle ABC, $a = 12$, $b = 5$ and $\tan \frac{1}{2}(A + B) = \dfrac{2}{7}$. Find $\tan \frac{1}{2}(A - B)$.   17_____

18. Find the area of triangle ABC if $b = 7$, $c = 12$ and $A = 30°$.   18_____

19. In triangle *ABC*, if sin $(A + B) = 3/5$, what is the value
of sin *C*?   19_____

20. As angle *x* increases from $0°$ to $360°$, tan *x* increases in
(a) no quadrants   (b) the first and third quadrants only
(c) the second and fourth quadrants only   (d) all four quadrants
[Write *a*, *b*, *c* or *d* to indicate which correctly completes the state-
ment.]   20_____

Part II

Answer three questions from this part. Show all work unless otherwise directed.

21. Find all values of *x* between $0°$ and $360°$ that satisfy the equation
$6 \cos^2 x + 5 \sin x = 2$.   [10]

22. *a Sketch* the graph of the equation $y = \sin 2x$ from $x = 0$ to $x = 2\pi$
radians.   [6]

b On the same set of axes used in part *a*, *draw* the graph of $y = \frac{1}{2}$.   [2]

c From the graphs made in answer to parts *a* and *b*, determine the number
of values of *x* between 0 and 2π radians that satisfy the equation
$\sin 2x = \frac{1}{2}$.   [2]

23. *a* Starting with the formulas for sin $(A + B)$ and cos $(A + B)$, *derive*
the formula for tan $(A + B)$ in terms of tan *A* and tan *B*.   [5]

b Derive the formula for tan $2A$ in terms of tan *A*.   [2]

c Using the formula derived in part *a*, prove that tan $(180° + B) =$
tan *B*.   [3]

24. *a* Prove the identity $\dfrac{\sec x + \csc x}{\tan x + \cot x} = \sin x + \cos x$.   [5]

b Prove the identity $\dfrac{\sin (A + B) + \sin (A - B)}{\sin (A + B) - \sin (A - B)} = \dfrac{\tan A}{\tan B}$.   [5]

25. Unless otherwise directed, answers to this question may be left in terms
of π.

a Express in degrees an angle of $\dfrac{3\pi}{2}$ radians. [2]

b Express in radians an angle of $660°$.   [2]

c A rod, pivoted at one end, rotates through $\dfrac{2\pi}{3}$ radians. If the rod is 6
inches long, how many inches does the free end travel?   [3]

d A wheel rotates at the rate of 165 revolutions per minute. Express in
radians the angle through which it rotates in one *second*.   [3]

Part III

Answer two questions from part III.

26. In triangle *ABC*, $a = 22.4$, $b = 24.8$ and $c = 18.6$. Find angle *B* to the *nearest degree.*   [10]

27. *a* Show that the area *K* of a regular pentagon whose apothem is *a* is given by the formula $K = 5a^2 \tan 36°$.   [5]

    b Using the formula in part *a*, find, to the *nearest tenth*, the apothem of a regular pentagon whose area is 431.   [5]

28. Just as a plane flies over a level straight line segment joining two ground observation posts, 4,680 feet apart, it is spotted by observers at both posts. If the angles of elevation of the plane from the two posts at this moment are 72° 20′ and 51° 50′, respectively, find, to the *nearest ten feet*, the height at which the plane is flying.   [10]

29. A triangular plot *MPQ* is surveyed. The direction of *P* from *Q* is N 22° 20′ W, and the direction of *M* from *Q* is N 82° 40′ W. If $QP = 47.6$ feet and $QM = 38.8$ feet, respectively, find angle *M* to the *nearest ten minutes*.   [5, 5]