The University of the State of New York

321st High School Examination

TRIGONOMETRY

Thursday, June 24, 1954 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in trigonometry.

The minimum time requirement is four or five recitations a week for half a school year,

Answer five questions from parts II and III, including at least two questions from each part.

Part II

Answer at least two questions from part II. All work, including computation, should be shown.

21 Find all values of x greater than 0° and less than 360° that satisfy the following equation: [Express approximate values of x to the *nearest degree.*] [10]

$$7\cos^2 x - 4\sin x = 4$$

22 *a* Starting with formulas for $\sin(x+y)$ and $\cos(x+y)$, derive the formula for $\tan(x+y)$ in terms of $\tan x$ and $\tan y$. [5]

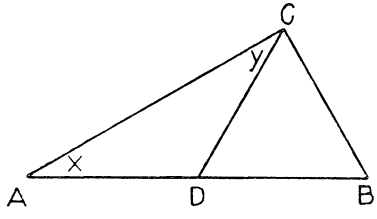
b Prove the identity: $\dfrac{2\cos x}{\cot x \cos x - \sin x} = \tan 2x$ [5]

23 *a* Draw the graph of $y = \cos 2x$ as x varies from 0 to 2π radians at intervals of $\dfrac{\pi}{6}$. [6]

b Place on the graph the letter A to indicate the point used in finding the value of y for which x is equal to $\dfrac{\pi}{12}$. [2]

c From the graph made in answer to *a*, determine the number of values of x between 0 and 2π for which y is equal to 0.6. [2]

24 In triangle ABC, angle ACB is a right angle. Using the letters given on the figure, show that each of the following relationships is true:



a $BD = \dfrac{BC\cos y}{\sin(x+y)}$ [5]

b $BD = \dfrac{AB\sin x\cos y}{\sin(x+y)}$ [3]

c $BD = \frac{1}{2}AB$ if $x = y$ [Use the relationship given in *b*.] [2]

[OVER]

Part III

Answer at least two questions from part III. All work, including computation, should be shown.

25 In triangle ABC, $AB = 16.6$, $BC = 12.5$ and $AC = 22.7$. Find angle B to the *nearest degree.* [10]

26 The navigator of a ship sights a lighthouse bearing N 32° W. After the ship has sailed on course N 27° E for 3.4 miles, the navigator then finds that the bearing of the lighthouse is N 78° W. Find to the *nearest mile* the distance of the ship from the lighthouse at the time the second bearing was taken. [6, 4]

27 Two forces of 320 pounds and 265 pounds act on a body at an angle of 72° with each other. Find to the *nearest degree* the angle formed by the lines of action of the resultant and the larger force. [10]

28 $ABCD$ is a trapezoid with leg DA perpendicular to base AB. Base $DC = 9$, angle $C = 118°$ and diagonal BD makes an angle of 35° with AB. Find to the *nearest integer* the altitude of the trapezoid. [10]

Be sure you have answered a total of five questions from parts II and III.

Fill in the following lines:

Name of pupil.....Name of school.....

Part I

Answer all questions in part I. Each correct answer will receive 2½ credits. No partial credit will be allowed.

1 Find the value of $\tan \frac{\pi}{4}$ 1.....

2 Find the radius of a circle in which a central angle of 3 radians subtends an arc of 24 feet. 2.....

3 If $3 \cot A = 2$, express A as an inverse trigonometric function. 3.....

4 Find n if $\log n = 1.3807$. 4.....

5 Find to the *nearest minute* the positive acute angle whose cosine is 0.9403. 5.....

6 Find $\log \sin 54° 32'$. 6.....

7 Ship C is 100 miles due east of location A, and ship B is directly north of C and N 70° E from A. Find to the *nearest mile* the distance from B to C. 7.....

8 In triangle ABC, $a = 10$, $b = 8$ and $C = 54°$. Find to the *nearest integer* the area of the triangle. 8.....

9 In triangle ABC, $a = 8.4$, $b = 10$ and $\sin A = 0.42$. Find $\sin B$. 9.....

10 In triangle ABC, $a = 5$, $b = 8$ and $c = 10$. Find $\cos B$. 10.....

11 If $\cos \theta = \frac{5}{7}$, find $\cos 2\theta$. 11.....

12 If θ is a positive acute angle and $\cos \theta = a$, express $\sin \frac{\theta}{2}$ in terms of a. 12.....

13 Express $\sin (x - 30°)$ in terms of $\sin x$ and $\cos x$. [Answer may be left in radical form.] 13.....

[OVER]

Directions (14–20): Indicate the correct completion for *each* statement by writing the letter *a*, *b* or *c* on the line at the right.

14 Sin 280° equals (*a*) cos 10° (*b*) — cos 10° (*c*) — cos 80° 14.....

15 Cos 100° + cos 20° equals (*a*) cos 40° (*b*) sin 40° (*c*) $\sqrt{3}$ cos 40° 15.....

16 If tan θ is negative and cos θ is positive then θ lies in the (*a*) second quadrant (*b*) third quadrant (*c*) fourth quadrant 16.....

17 The maximum value of 3 sin $2x$ is (*a*) 1 (*b*) 3 (*c*) 6 17.....

18 In triangle ABC if sin $A = x$ and if cos $B = x$, then the angle $(A + B)$ is (*a*) acute (*b*) right (*c*) obtuse 18.....

19 Using the data $A = 44° 30'$, $a = 15$ and $b = 20$, (*a*) only one triangle can be constructed (*b*) two triangles can be constructed (*c*) it is not possible to construct a triangle 19.....

20 As x varies from $\frac{\pi}{2}$ to $\frac{3\pi}{2}$ radians, the graphs of $y = \tan x$ and $y = \sin \frac{x}{2}$ when drawn on the same set of axes intersect in (*a*) one point (*b*) two points (*c*) three points 20.....

FOR TEACHERS ONLY

INSTRUCTIONS FOR RATING
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Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use check marks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow $2\frac{1}{2}$ credits for each correct answer; allow no partial credit. For questions 14–20, allow credit if the pupil has written the correct answer instead of the letter *a, b* or *c*.

(1) 1
(2) 8
(3) $A = \cot^{-1}\frac{2}{3}$
(4) 24.03
(5) 19° 54′
(6) 9.9109 — 10
(7) 36
(8) 32
(9) $\frac{1}{2}$
(10) $\frac{61}{100}$
(11) $\frac{1}{49}$
(12) $\sqrt{\frac{1-a}{2}}$
(13) $\frac{\sqrt{3}}{2}\sin x - \frac{1}{2}\cos x$
(14) *b*
(15) *a*
(16) *c*
(17) *b*
(18) *b*
(19) *b*
(20) *a*