TRIGONOMETRY

Thursday, June 18, 1953 —9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (a) name of school where you have studied, (b) number of weeks and recitations a week in trigonometry.

The minimum time requirement is four or five recitations a week for half a school year, or the equivalent.

Answer five questions from parts II and III, including at least two questions from each part.

Part II

Answer at least two questions from part II.

21 Find all positive values of A less than 360° which satisfy the equation $5 \cos^2 A - \sin A = 1$. [Express approximate values of A to the *nearest degree*.]   [10]

22 a Starting with the formula for $\cos (x + y)$, derive the formula for $\cos 2x$ in terms of $\cos x$. [4]

b Prove the identity: $\dfrac{2 \tan x - \sin 2x}{2 \sin^2 x} = \tan x$ [6]

23 a On the same set of axes sketch the graphs of $y = \tan x$ and $y = \cos \dfrac{x}{2}$ as x varies from 0 to 2π radians.   [4, 4]

b From the graphs drawn in answer to (a), determine the number of values of x between 0 and 2π radians which satisfy the equation $\tan x = \cos \dfrac{x}{2}$.   [2]

24 In quadrilateral $ABCD$, DA and CB are perpendicular to AB. Using the letters indicated on the figure, show that $BC = DC \tan x \sin y$.   [10]
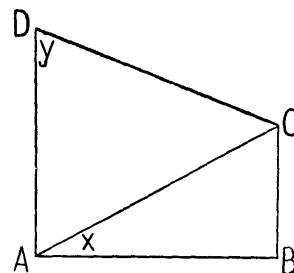
[1]   [OVER]

Part III
Answer at least two questions from part III.

25 In triangle ABC, $a = 18.6$, $b = 22.9$ and $c = 12.7$. Find B to the *nearest degree*.   [10]

26 A straight road is to be built from A to B, two points separated by woods. Point C, easily accessible to both A and B, is chosen. It is found that CA is 960 yards, CB is 1230 yards and angle ACB is 76°. Find to the *nearest degree* angle CAB.   [4, 6]

27 From mountain peak P directly above straight tunnel RS, the angles of depression of the ends of the tunnel, R and S, are 43° and 52°, respectively. If distance PR is 3710 feet and the tunnel slopes upward from R at an angle of 6° with the horizontal, find the length of the tunnel to the *nearest ten feet*.   [6, 4]

28 Two lighthouses A and B bear N 40° E from ship Q, A being nearer Q. After the ship has sailed for one hour on a course S 50° E to position R, A is due north of R, and B bears N 9° E from R. If the distance between A and B is 8.3 miles, find to the *nearest tenth of a mile* distance QR.   [10]

Fill in the following lines:

Name of pupil..Name of school...

Part I

Answer all questions in part I. Each correct answer will receive 2½ credits. No partial credit will be allowed.

1 Find the value of the cosecant of 135°. [Answer may be left in radical form.]

1...................

2 Find the number of radians in an angle of 210°. [Answer may be left in terms of π.]

2...................

3 A central angle of 1.5 radians intercepts an arc of 6.0 inches. Find the radius of the circle.

3...................

4 If $2\theta = \cos^{-1} \frac{1}{2}$, find the acute angle θ.

4...................

5 Find the logarithm of 1.568

5...................

6 Find cot 41° 23′

6...................

7 Find to the *nearest minute* the acute angle A if log sin $A = 9.9226 - 10$

7...................

8 In triangle ABC, $a = 15$, $b = 6$ and $A = 30°$. Find sin B.

8...................

9 In triangle ABC, $b = 7$, $c = 4$ and $\cos A = \frac{3}{4}$. Find a.

9...................

10 In triangle ABC, $a = 12$, $b = 8$ and $A + B = 84°$. Find $\tan \dfrac{A - B}{2}$ to the *nearest hundredth*.

10...................

11 If a road rises 32 feet in a horizontal distance of 400 feet, find to the *nearest degree* the angle that the road makes with the horizontal.

11...................

12 If $\tan A = -\frac{3}{4}$, find tan $(180° - A)$.

12...................

13 Express tan $(x + y)$ in terms of tan x and tan y.

13...................

[3]

[OVER]

14 If x is a positive acute angle and sin $x = \dfrac{2}{\sqrt{13}}$, find sin $2x$. 14................

15 If x is a positive acute angle and cos $x = \tfrac{1}{8}$, find cos $\tfrac{1}{2}$ x. 15................

16 If x is a positive acute angle, express cot x in terms of cos x. 16................

Directions (17–20) : Indicate the correct answer or completion for *each* of the following by writing on the line at the right the letter *a*, *b* or *c*.

17 Which of the following is equal to sin 40°? (a) sin 220° (b) sin 130° (c) cos (—50°) 17......

18 Sin 32° + sin 28° equals (a) $\sqrt{3}$ sin 2° (b) $\sqrt{3}$ cos 2° (c) cos 2° 18......

19 Using the data, $A = 42°, a = 6, b = 10$, it is possible to construct (a)only one triangle (b)two different triangles (c)no triangle 19......

20 As x varies from 0 to π radians, the graphs of $y = 2$ and $y = 2$ sin x when drawn on the same set of axes (a)intersect in distinct points (b)are tangent (c)have no points in common 20......