The University of the State of New York

315TH HIGH SCHOOL EXAMINATION

TRIGONOMETRY

Thursday, June 19, 1952 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in trigonometry.

The minimum time requirement is four or five recitations a week for half a school year, or the equivalent.

Answer five questions from parts II and III, including at least two questions from each part.

Part II

Answer at least two questions from part II.

21 Find all positive values of A less than 360° which satisfy the equation $4 \sin A + 1 = 3 \csc A$. [Express approximate values of A to the *nearest degree.*] [10]

22 *a* Starting with the formulas for $\sin (x - y)$ and $\cos (x - y)$, derive the formula for $\tan (x - y)$ in terms of $\tan x$ and $\tan y$. [5]

b Prove that the following equation is an identity: [5]

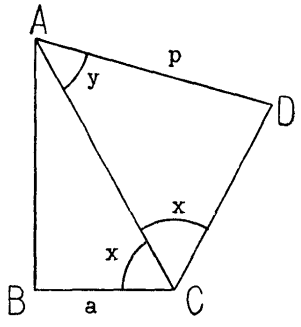
$$\frac{\sin x + \sin 2x}{\sec x + 2} = \frac{\sin x}{\sec x}$$

23 *a* Draw the graph of $y = \sin \frac{x}{2}$ as x varies from 0 to 2π radians at intervals of $\frac{\pi}{3}$ [6]

b On the set of axes used in answer to *a*, draw the graph of $y = .7$ [2]

c By means of capital letters, indicate on the graphs made in answer to *a* and *b* the points whose abscissas satisfy the equation $\sin \frac{x}{2} = .7$ [2]

24 In quadrilateral $ABCD$, CA bisects $\angle BCD$ and AB is perpendicular to BC. Using the letters indicated on the figure, show that



(1) $p = \frac{AC \sin x}{\sin (x + y)}$ [6]

(2) $p = \frac{a \tan x}{\sin (x + y)}$ [4]

[OVER]

Part III

Answer at least two questions from part III.

25 The sides of a triangle are 140, 246 and 318. Find to the *nearest degree* the largest angle of the triangle. [10]

26 A building and a monument stand on the same horizontal plane. The height of the monument is 462 feet. From a window of the building the angle of elevation of the top of the monument is 34° and the angle of depression of its base is 53°. Find to the *nearest foot* the distance from the building to the monument. [5, 5]

27 An airplane carrier is steaming N 36° W at 24 miles per hour. A plane takes off from the carrier and flies 276 miles due west. In what direction must the plane then fly in order to reach the carrier 3 hours after taking off? [Express your answer to the *nearest degree.*] [5, 5]

28 A straight road at right angles to the shore line of a river rises at an angle of 15° to the horizontal. At a point on this road 156 yards back from the shore line, the angle of depression of a point directly across on the opposite edge of the river is 6° 40′. Find to the *nearest yard* the width of the river. [5, 5]

Fill in the following lines:

Name of pupil..... Name of school.....

Part I

Answer all questions in part I. Each correct answer will receive 2½ credits. No partial credit will be allowed.

1 Express in radians an angle of 200°. 1.....

2 Express cos 156° as a function of a positive acute angle. 2.....

3 If $2 \tan A = 5$, express A as an inverse function. 3.....

4 A pendulum 12 inches long swings through an arc 8 inches long. Find the number of radians in the angle through which it swings. 4.....

5 Find the logarithm of 150.3 5.....

6 Find log cos 56° 36′ 6.....

7 Find to the *nearest minute* the positive acute angle whose tangent is .7893 7.....

8 In triangle ABC, $C = 90°$. If $\sec A = \frac{6}{5}$ and $c = 42$, find b. 8.....

9 In triangle ABC, $a = 5$, $b = 7$ and $c = 9$. Find $\cos B$. 9.....

10 In triangle ABC, $A = 37°$, $b = 20$ and $c = 10$. Find to the *nearest integer* the area of the triangle. 10.....

11 In triangle RST, $R = 105°$ and $S = 15°$. Find the value of $\frac{r+s}{r-s}$. [Answer may be left in radical form.] 11.....

12 In triangle RST, $r = 6$, $s = 9$ and $\sin R = \frac{1}{5}$. Find $\sin S$. 12.....

13 If x is an acute angle and $\cos x = a$, express $\cos \frac{x}{2}$ in terms of a. 13.....

14 If $\tan x = 4$, find the value of $\tan 2x$. 14.....

15 Is $-60°$ a root of the equation $2 \sin \theta + \sqrt{3} = 0$? [Answer *yes* or *no.*] 15.....

16 If $\sin x = \frac{5}{\sqrt{34}}$ and $\cos x$ is negative, find $\tan x$. 16.....

Directions (17–20): Indicate the correct completion for each of the following by writing on the line at the right the letter *a*, *b* or *c*.

17 Sin 80° — sin 10° equals (*a*) sin 70° (*b*) $\sqrt{2}$ sin 35° (*c*) $\sqrt{2}$ cos 35° 17.....

18 $\text{Sin}^2 3x + \cos^2 3x = 1$ is true for (*a*) no values of x (*b*) only certain values of x (*c*) all values of x 18.....

19 The minimum value of $3 \sin 2x$ is (*a*) —1 (*b*) —3 (*c*) —6 19.....

20 As x varies from 0 to π radians, the graphs of the functions $y = \tan x$ and $y = \cos x$ intersect in (*a*) 1 point (*b*) 2 points (*c*) 3 points 20.....