The University of the State of New York

309TH HIGH SCHOOL EXAMINATION

TRIGONOMETRY

Thursday, June 22, 1950 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in trigonometry.

The minimum time requirement is four or five recitations a week for half a school year, or the equivalent.

Answer five questions from parts II and III, including at least two questions from each part.

Part II

Answer at least two questions from part II.

21 *a* Prove the identity: $\cos 2A = \dfrac{1 - \tan^2 A}{1 + \tan^2 A}$ [7]

b Beginning with the formula for $\tan (x + y)$, derive the formula for $\tan 2x$. [3]

22 *a* On the same set of axes sketch the graphs of $y = \tan x$ and $y = 2 \cos x$ from 0 to 2π radians inclusive. [3, 5]

b From the graphs made in answer to *a*, determine the number of values of x between 0 and 2π radians that satisfy the equation $\tan x = 2 \cos x$. [2]

23 Find all values of x between 0° and 360° which satisfy the equation $2 \cos^2 \frac{1}{2}x = \sin^2 x$. [10]

24 Two towers whose heights are a and b (b being greater than a) stand on level ground. The angle of elevation of the top of the shorter tower from the foot of the taller tower is y and the angle of elevation of the top of the taller tower from the foot of the shorter tower is x.

a Show that $x = \tan^{-1}\left(\dfrac{b \tan y}{a}\right)$ [6]

b Find x to the *nearest degree* if $b = 120$, $a = 50$ and $y = 35°$. [4]

Part III

Answer at least two questions from part III.

25 In triangle ABC, $a = 316$, $b = 227$ and $C = 76° 20'$. Find A to the *nearest minute*. [10]

26 In a certain air race, the course was a triangle with sides 155 miles, 212 miles and 307 miles. Find, to the *nearest degree,* the angle at the turn between the 155-mile and 307-mile sides. [10]

27 Two lighthouses, A and B, are each directly north of a ship, A being the lighthouse nearer the ship. After the ship has proceeded 28 miles on a course N 55° E, A bears directly west and B, N 40° W. Find, to the *nearest mile,* the distance between A and B. [5, 5]

28 Two forces are to act on a body to produce a resultant of 74 pounds. If the lines of action of the two forces form an angle of 65° and one of the forces is 45 pounds, find, to the *nearest pound,* the other force. [4, 6]

Fill in the following lines:

Name of pupil.....Name of school .....

Part I

Answer all questions in part I. Each correct answer will receive 2½ credits. No partial credit will be allowed.

1 Express in degrees an angle of $\frac{2\pi}{9}$ radians. 1.....

2 Express cos 224° as a function of a positive acute angle. 2.....

3 Find the smallest positive value of x for which $\sin^2 x = \frac{1}{2}$. 3.....

4 If x is an acute angle and $\sin x = \frac{2}{\sqrt{29}}$, find cot x. 4.....

5 If $\cos x = \frac{1}{9}$ and x is a positive acute angle, find $\sin \frac{1}{2}x$. 5.....

6 If $A = \cos^{-1} \frac{5}{13}$ and A is in the first quadrant, find cot A. 6.....

7 Find sin 39° 16′. 7.....

8 Find, to the *nearest minute,* the positive acute angle A for which log cot $A = 9.8306 - 10$. 8.....

9 Find the number whose logarithm is 3.3914. 9.....

10 Express cos 70° + cos 50° as a function of 10°. 10.....

11 Express cos A in terms of tan A where A is an angle in the first quadrant. 11.....

12 In triangle ABC, $a = 9$, $b = 5$, $c = 8$; find cos B. 12.....

13 In triangle ABC, $a = 12$, $\sin A = \frac{1}{2}$, $\sin C = \frac{1}{4}$; find c. 13.....

14 In triangle ABC, $a = 10$, $b = 6$, $C = 58°$. Find, to the *nearest hundredth,* $\tan \frac{1}{2}(A-B)$. 14.....

15 As x varies from 180° to 360°, which function of x, other than the tangent, increases throughout this interval? 15.....

16 Find the maximum value of $\sin \frac{1}{2}x$. 16.....

Directions (questions 17–20) — Indicate whether *each* statement is true or false by writing *true* or *false* on the line at the right.

17 In triangle ABC, in which $C = 90°$, $\tan B = \cot A + \cos C$. 17.....

18 $\sin 3A \cos A + \cos 3A \sin A = \sin 4A$. 18.....

19 $\tan (-A) = \frac{\sin (-A)}{\cos A}$ 19.....

20 One of the values of x for which $\tan (x + y)$ is equal to $\frac{1 + \tan y}{1 - \tan y}$ is 225°. 20.....