The University of the State of New York

306TH HIGH SCHOOL EXAMINATION

TRIGONOMETRY

Thursday, June 23, 1949 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in trigonometry.

The minimum time requirement is four or five recitations a week for half a school year, or the equivalent.

Answer five questions from parts II and III, including at least two questions from each part.

Part II

Answer at least two questions from part II.

21 *a* Starting with a formula for $\cos 2A$, derive the formula for $\sin \frac{1}{2}x$ in terms of $\cos x$. [5]

b Prove that the following equality is an identity:

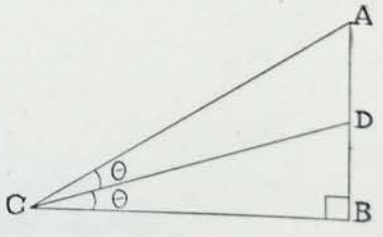
$$\frac{1 + \sec x}{\csc x} = \sin x + \tan x \quad [5]$$

22 *a* On the same set of axes sketch the graphs of $y = 2 \sin x$ and $y = \cos x$ from 0 to 2π radians inclusive. [5, 3]

b From the graphs made in answer to *a*, determine the quadrants in which are found the angles that satisfy the equation $2 \sin x = \cos x$ [2]

23 Find to the *nearest degree* all values of x between $0°$ and $360°$ which satisfy the equation $3 \cos 2x - 7 \cos x = 0$ [10]

24 Using the diagram at the right in which AB is perpendicular to CB show that $DA = \frac{CB \tan \theta}{\cos 2\theta}$ [10]



Part III

Answer at least two questions from part III.

25 In triangle ABC, $a = 386$, $b = 514$, $C = 52° 20'$. Find A to the *nearest minute.* [10]

26 A tree is growing at the bank of a river. The angle of elevation of the top from a point directly across on the other bank is $37° 50'$. From a second point 125 feet from the first and in line with the first point and the tree, the angle of elevation of the top of the tree is $22° 30'$. Find to the *nearest foot* the width of the river. [10]

27 A body is acted upon by two forces of 85 pounds and 215 pounds. If their resultant is 242 pounds, find to the *nearest minute,* the angle formed by the lines of action of the forces. [10]

28 A field has the form of a triangle in which two angles are $36° 10'$ and $64° 20'$ and the included side is 58 rods. Find the area of the field to the *nearest tenth* of an acre. [1 acre = 160 square rods.] [10]

Fill in the following lines:

Name of pupil.....Name of school.....

Part I

Answer all questions in part I. Each correct answer will receive 2½ credits. No partial credit will be allowed.

1 Find to the *nearest tenth* the number whose logarithm is 2.6687 1.....

2 Find the value of cos 73° 22′ 2.....

3 Find acute angle A to the *nearest minute* if log tan $A = 9.6324 - 10$ 3.....

4 If $\log x = \log a + \log b$, express x in terms of a and b. 4.....

5 Find the value of tan $\frac{1}{2}x$ if x is the acute angle whose cosine is $\frac{3}{5}$. 5.....

6 Express cos A in terms of cot A if A is a positive acute angle. 6.....

7 Find the number of degrees in $\frac{5\pi}{6}$ radians. 7.....

8 In a circle whose radius is r, a central angle intercepts an arc whose length is $2r$. Find the number of radians in the angle. 8.....

9 Find the value of tan (—45°) 9.....

10 The pilot of a scouting plane left an airport and flew 200 miles in the direction N 35° E. Find, to the *nearest mile*, how far east of the airport he had flown. 10.....

11 If tan $x = a$ and tan $y = b$, express tan $(x + y)$ in terms of a and b. 11.....

12 Find the positive value of sin $(\cos^{-1} \frac{5}{13})$. 12.....

13 The sides of a triangle are 6, 4 and 3. Find the cosine of the smallest angle. 13.....

14 In triangle ABC, $a = 6$, $c = 5$ and $B = 60°$. Find the value of tan $\frac{1}{2}(A - C)$. [Answer may be left in radical form.] 14.....

15 Two sides of a triangle are a and b and their included angle is C. Express the area of the triangle in terms of a, b and C. 15.....

16 In triangle ABC, $A = 30°$ and $B = 45°$. Find the value of $\frac{a}{b}$. 16.....

Directions (questions 17–20) — Indicate whether *each* statement is true or false by writing *true* or *false* on the line at the right.

17 The minimum value of sin $3x$ is —3. 17.....

18 $\text{Cos}^2 3A = \dfrac{1 + \cos 6A}{2}$ 18.....

19 As x varies from 0° to 180°, csc x decreases and then increases. 19.....

20 Triangle ABC is determined if the given parts are angle A, side a and side b and if a is greater than b. 20.....