The University of the State of New York

303D HIGH SCHOOL EXAMINATION

TRIGONOMETRY

Thursday, June 24, 1948 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II, III and IV (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in trigonometry.

The minimum time requirement is four or five recitations a week for half a school year, or the equivalent.

Answer five questions from parts II, III and IV, including at least one question from each part.

Part II

Answer at least one question from part II.

21 *a* Starting with the formulas for $\sin (A + B)$ and $\cos (A + B)$, derive the formula for $\tan (A + B)$ in terms of $\tan A$ and $\tan B$. [7]

b Using the formula derived in answer to *a*, derive the formula for $\tan 2A$. [3]

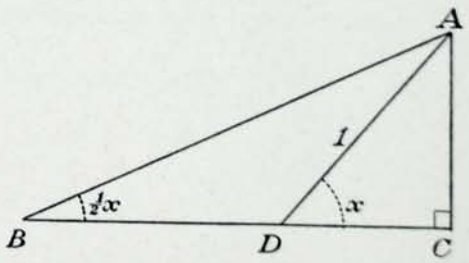
22 *a* Using the same set of axes, sketch the graphs of $y = \cos x$ and $y = \tan x$ as x varies from 0 to 2π radians. [8]

b Indicate on the graphs made in answer to *a* the points whose abscissas give solutions of the equation $\cos x = \tan x$. [2]

23 *a* Solve for all values of x between 0° and 360° : $2 \sin^2 x - \cos 2x = 0$ [7]

b Express $\cos A \cot A$ in terms of $\sin A$. [3]

24 Using the drawing in which $AD = 1$, $AC \perp BC$, $\angle ABC = \frac{1}{2}x$ and $\angle ADC = x$, show that $\tan \frac{1}{2}x = \dfrac{\sin x}{1 + \cos x}$ [10]



Part III

Answer at least one question from part III.

25 In triangle ABC, $a = 30$, $c = 54$, $B = 94° 40'$. Find C to the *nearest minute.* [10]

26 A surveyor running a line due east reached the edge of a swamp. He then ran a line 225 feet in the direction S 47° 30′ E and from the point thus reached, he ran a line in the direction N 52° 10′ E. How far had he continued on this line when he reached a point on the original line extended? [Express your answer to the *nearest foot.*] [10]

27 The sides of a triangular lot are 521 feet, 467 feet and 208 feet. Find to the *nearest minute* the angle opposite the longest side. [10]

Part IV

Answer at least one question from part IV.

28 An airplane leaves Chicago (latitude 41° 50′ N, longitude 87° 40′ W) and flies to Paris (latitude 48° 50′ N, longitude 2° 20′ E). Find in nautical miles the distance traveled. [10]

29 In spherical triangle ABC, $a = 36° 20'$, $b = 72° 30'$ and $C = 82°$. Find side c to the *nearest degree.* [10]

Fill in the following lines:

Name of pupil.....Name of school.....

Part I

Answer all questions in part I. Each correct answer will receive 2½ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1 Find the logarithm of .04732 1.....

2 If log sin $A = 9.8072 - 10$, find acute angle A to the *nearest minute*. 2.....

3 Find the value of cos 75° 34′ 3.....

4 In triangle ABC, $b = 15$, $c = 20$ and $A = 19°$. Find to the *nearest integer* the area of triangle ABC. 4.....

5 In triangle ABC, $A = 45°$ and $B = 30°$. Find $\frac{a}{b}$. [Answer may be left in radical form.] 5.....

6 In triangle ABC, $a = 2$, $b = 3$ and $c = 4$. Find the *cosine* of the smallest angle. 6.....

7 Express tan 140° as a function of a positive acute angle. 7.....

8 Is 45° a value of x which satisfies the equation $\sin 3x + \sin 2x - \sin x = 0$? [Answer *yes* or *no*.] 8.....

9 Express $\sin x + \sin y$ as a product of two functions. 9.....

10 Cosine $A = -\frac{4}{5}$ and sin A is positive. Find tan A. 10.....

11 A central angle in a circle whose radius is 4 inches intercepts an arc of 8 inches. How many radians are there in the angle? 11.....

12 From the data $A = 40°$, $a = 15$, $b = 13$, there can be constructed (*a*) no triangle (*b*) one triangle (*c*) two triangles. Which is correct (*a*), (*b*) or (*c*)? 12.....

13 If angle A is acute, express sin A in terms of tan A. 13.....

14 Given csc $70° = a$, express sin 110° in terms of a. 14.....

15 Express tan $(45° - x)$ in terms of tan x. 15.....

16 In right spherical triangle ABC, $C = 90°$ and angle A and angle B are known. Write the formula which should be used to find a. 16.....

Directions (questions 17–20) — Indicate whether each statement is true or false by writing the word *true* or *false* on the line at the right.

17 If the hypotenuse of a right spherical triangle is greater than 90° and side a is less than 90°, then side b is less than 90°. 17.....

18 The maximum value of $3 \cos 4x$ is 4. 18.....

19 $\frac{\cos(-A)}{\sin A} = \cot A$ 19.....

20 $\cos 10B = \cos^2 5B - \sin^2 5B$ 20.....