TRIGONOMETRY

Thursday, June 19, 1947 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II, III and IV (a) name of school where you have studied, (b) number of weeks and recitations a week in trigonometry.

The minimum time requirement is four or five recitations a week for half a school year, or the equivalent.

Answer five questions from parts II, III and IV, including at least one question from each part.

Part II

Answer at least one question from part II.

21 *a* Starting with the formula for cos $2A$, derive the formula for cos $\dfrac{x}{2}$ in terms of cos x. [5]

 b Starting with the formulas for sin $(x - y)$ and cos $(x - y)$, derive the formula for tan $(x - y)$. [5]

22 *a* Show that the following statement is true for all values of A:
$$\sec^2 A + \csc^2 A = \sec^2 A \csc^2 A \qquad [4]$$

 b Solve the equation tan $2x = \cot x$ for the smallest positive value of x. [6]

23 *a* Sketch the graph of $y = \sin 2x$ as x varies from $0°$ to $360°$ inclusive. [5]

 b On the same set of axes used in answer to *a*, sketch the graph of $y = \cos x$ as x varies from $0°$ to $360°$ inclusive. [3]

 c From the graphs made in answer to *a* and *b*, determine the *number of values* of x between $0°$ and $360°$ for which sin $2x$ equals cos x. [2]

24 *a* If the apothem of a regular polygon of n sides is a and an interior angle is 2θ, derive a formula for the area of the polygon in terms of n, a and θ. [6]

 b Using the formula derived in answer to *a*, find, correct to the *nearest integer*, the area of a regular octagon whose apothem is 2.4. [4]

[1]

Part III

Answer at least one question from part III.

25 A triangular lot has sides 62.2 feet, 75.3 feet and 100.5 feet. Find the area correct to the *nearest square foot.* [10]

26 Navigation lights A and B are situated 1500 feet apart on a straight shore line. B lies directly north of A. The bearing of a boat from light A is N 40° E and from light B, S 60° E.

    a Find, correct to the *nearest foot*, the distance of the boat from A. [6]

    b Find, correct to the *nearest foot*, the distance from the boat to the nearest point on shore. [4]

27 Given triangle ABC with $a = 42$, $b = 24$, and $C = 64°$. Find angle A correct to the *nearest minute.* [10]

Part IV

Answer at least one question from part IV.

28 Given spherical triangle ABC in which $C = 90°$, $B = 124°$ and $c = 82°$. Find a correct to the *nearest minute.* [10]

29 A pilot flies the great circle course between an island off Bermuda (Lat. 32° 20′ N, Long. 64° 50′ W) and New York City (Lat. 40° 43′ N, Long. 74° 0′ W). The distance between the two places is 670 nautical miles. Find, correct to the *nearest degree*, the bearing of New York City from this island. [10]

Fill in the following lines:

Name of school................................Name of pupil................................

Part I

Answer all questions in part I. Each correct answer will receive 2½ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1 Find log cos 28° 33'

1................

2 Find, correct to the *nearest minute*, the acute angle whose tangent is .6432

2................

3 Find, correct to the *nearest hundredth*, the number whose logarithm is 1.3799

3................

4 Express cos 140° as a function of a positive angle less than 45°.

4................

5 Express $\frac{4\pi}{3}$ radians in degrees.

5................

6 In triangle ABC, $A = 30°$, $B = 45°$ and $a = 4$. Find b. [Answer may be left in radical form.]

6................

7 In triangle ABC, $A = 60°$, $c = 4$ and $b = 3$. Find a. [Answer may be left in radical form.]

7................

8 In triangle ABC, $A = 120°$ and $B = 30°$. Find, correct to the *nearest tenth*, the value of $(a + b)$ when $(a - b) = 1$

8................

9 Express cos A in terms of tan A if A is an angle in the first quadrant.

9................

10 Find the smallest positive angle greater than 0° which satisfies the equation $\sin^2 x - \sin x = 0$

10................

11 What is the value of cos π?

11................

12 In a right spherical triangle ABC in which C is the right angle, a and A are given. Write the formula that should be used to find c.

12................

Directions (questions 13–16) — Indicate whether *each* statement is true or false by writing the word *true* or the word *false* on the line at the right.

13 $\sin 2x = \pm \sqrt{\dfrac{1 - \cos 4x}{2}}$

13................

14 $\sin A + \sin B = 2 \sin \frac{1}{2}(A + B) \cos \frac{1}{2}(A - B)$

14................

15 As the sine of an angle decreases from 0 to -1, the tangent of that angle decreases.

15................

16 There is no triangle in which $B = 40°$, $b = 30$ and $a = 50$

16................

Directions (questions 17–20) — Indicate the correct answer to *each* question by writing on the line at the right the letter a, b or c.

17 Sin $(x + y)$ equals (a) sin x sin y — cos x cos y (b) sin x cos y — cos x sin y (c) sin x cos y + cos x sin y

17......

18 The greatest value of 4 sin 2x is (a) 1 (b) 4 (c) 8

18......

19 In right spherical triangle ABC in which $C = 90°$, $b = 85°$ and $a = 140°$, c is (a) less than 90° (b) equal to 90° (c) greater than 90°

19......

20 If in two polar triangles angle A of one triangle is opposite side a' of the other, then (a) $A = a'$ (b) $A + a' = 90°$ (c) $A + a' = 180°$

20......

[3]