TRIGONOMETRY

Thursday, June 20, 1946 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II, III and IV (a) name of school where you have studied, (b) number of weeks and recitations a week in trigonometry.

The minimum time requirement is five recitations a week for half a school year, or the equivalent.

Answer five questions from parts II, III and IV, including at least one question from each part.

Part II

Answer at least one question from part II.

21 a Starting with the formulas for sin $(x - y)$ and cos $(x - y)$, derive the formula for tan $(x - y)$.   [6]

 b If angle x and angle y are in the first quadrant, sin $x = \frac{3}{5}$ and cos $y = \frac{12}{13}$, find the value of tan $(x - y)$.   [4]

22 a Write the equation $\tan^2 x = \sec x$ in terms of cos x.   [2]

 b Using the equation obtained in answer to a, find the value of cos x, expressing the result in decimal form.   [5]

 c How many values of x are there between 0° and 360° which satisfy the given equation? Find *one* of these values correct to the *nearest degree*.   [1, 2]

23 Using the law of cosines, prove that if two medians of a triangle are equal, the triangle is isosceles.   [10]

[Suggestion: Given triangle ABC with medians AD and BE equal. Using triangles ADC and BEC, express AD and BE in terms of sides a and b and angle C of triangle ABC.]

24 a Sketch the graph of $y = \sin x$ as x varies from $-\pi$ to $+\pi$ radians.   [5]

 b On the graph drawn in answer to a, illustrate the fact that sin $(-x)$ is equal to $-\sin x$ for values of x between $+\pi$ and $-\pi$ radians.   [3]

 c Is the fact illustrated in answer to b true for all values of x?   [2]

[1]   [OVER]

Part III
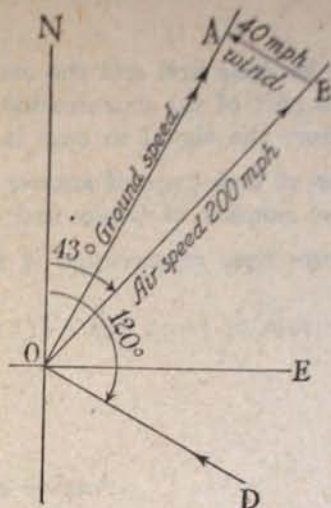### Answer at least one question from part III.

25 *a* From a point *d* feet above level ground, the angle of elevation of the top of a building is *x* and the angle of depression of the foot of the building is *y*. Show that the height *h* of the building is given by the formula $h = \dfrac{d (\tan x + \tan y)}{\tan y}$ [7]

b Find *h* correct to the *nearest foot* if *d* = 30 feet, *x* = 66° and *y* = 22° [3]

26 Radio station *A* is 312 miles directly north of station *B*. Shortly before noon both stations received signals from a ship *C* in distress. The bearing of *C* from *A* was found to be S 65° E and the bearing of *C* from *B* was N 75° E. If a plane left *B* at 12 noon, flying 220 mph, at what time did it reach *C*? [10]

27 The heading (angle *NOB*) of an airplane is 043° and its air speed (*OB*) is 200 mph. The wind is blowing from 120° (angle *NOD*) with a velocity (*BA*) of 40 mph. Find, correct to the *nearest degree*, the track (angle *NOA*) of the plane. [10]

Part IV
Answer at least one question from part IV.

28 Sides *AB* and *CB* of spherical triangle *ABC* are each 65° and angle *B* = 40°. Find, correct to the *nearest minute*, the great circle arc from *B* to the mid-point of *AC*. [10]

29 Find, correct to the *nearest nautical mile*, the distance from Honolulu (Lat. 21° 20′ N, Long. 157° 52′ W) to Midway Island (Lat. 28° 10′ N, Long. 177° 22′ W). [10]

Fill in the following lines:

Name of school...Name of pupil...

Part I

Answer all questions in part I. Each correct answer will receive 2½ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1 Express tan 250° as a function of a positive angle less than 45°.   1......................

2 The statement $\cos(-A) = \cos A$ is (a) true for all values of A, (b) true for only certain values of A or (c) not true for any value of A. Which is correct, (a), (b) or (c)?   2......................

3 Find, correct to the *nearest minute,* the positive acute angle whose cosine is .4325   3......................

4 Find log sin 138° 46′   4......................

5 In triangle ABC, $A = 45°$, $B = 30°$ and $b = 10$. Find a correct to the *nearest integer.*   5......................

6 In triangle ABC, $b = 4$, $c = 5$ and $\cos A = -\frac{1}{8}$. Find a.   6......................

7 The bearing of point B from point A is N 65° 30′ W. If B is 580 feet from A, how far west, correct to the *nearest foot,* is B from A?   7......................

8 Express $(1 + \sec A)(1 - \cos A)$ in terms of $\sin A$ and $\cos A$.   8......................

9 Find the value of x greater than 0° and less than 90° which satisfies the equation $2 \sin^2 x - \sqrt{3} \sin x = 0$ 9......................

10 How many values of x are there between 0° and 360° which satisfy the equation $2 \tan^2 x - 1 = 0$?   10......................

11 How many times does the graph of $2 \sin x$ cross the graph of $\cos x$ as x increases from 0 radians to π radians?   11......................

12 The sides of a central angle of $1\frac{1}{2}$ radians intercept an arc whose length is 6 inches. Find, in inches, the radius of the circle.   12......................

13 If $\cos x = \frac{4}{5}$ and x is in the first quadrant, find the value of $\sin 2x$.   13......................

14 If $\sin x = \frac{2}{3}$, find the value of $\cos 2x$.   14......................

15 If angle x is acute and $\cos x = \frac{4}{5}$, find the value of $\tan \dfrac{x}{2}$ 15......................

16 The expression $\dfrac{\sin A + \sin B}{\cos A + \cos B}$ is equal to (a) $\tan(A + B)$, (b) $\tan \frac{1}{2}(A + B)$ or (c) $\tan A + \tan B$. Which is correct, (a), (b) or (c)?   16......................

17 Is the following statement *true* or is it *false*? If, in plane triangle ABC, a is less than b but greater than $b \sin A$, then B must be acute.   17......................

18 In right spherical triangle ABC in which C is the right angle, B and b are known. Write the formula which should be used to find a.   18......................

19 In right spherical triangle ABC in which C is the right angle, $B = b = 30°$; find the sum of A, B and C.   19......................

20 Is there a right spherical triangle in which $a = 100°$, $b = 105°$ and $c = 95°$? Answer *yes* or *no*.   20......................

[3]