The University of the State of New York

294TH HIGH SCHOOL EXAMINATION

TRIGONOMETRY

Thursday, June 21, 1945 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II, III and IV (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in trigonometry.

The minimum time requirement is five recitations a week for half a school year, or the equivalent.

Answer five questions from parts II, III and IV, including at least one question from each part.

Part II

Answer at least one question from part II.

21 Solve, correct to the *nearest minute*, the equation $5 \cos^2 x + 13 \sin x - 11 = 0$ for all values of x between 0° and 360°. [10]

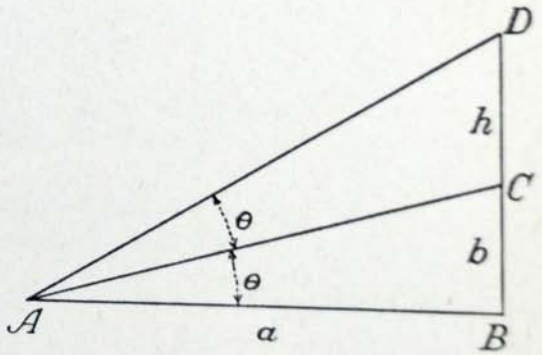
22 Derive the formula for $\cos(x + y)$ in which x, y and $x + y$ are positive acute angles. [10]

23 *a* Draw and letter clearly the line values of the sine, cosine and tangent of an angle in the second quadrant. [1, 1, 2]

b Sketch on one set of axes the graphs of $\sin x$, $2 \sin x$ and $\frac{1}{2} \sin x$ as x varies from 0 to π radians. [2, 2, 2]

24 A person standing at a distance a from a tower surmounted by a spire observes the tower and the spire to subtend the same angle θ, as shown in the drawing. If b is the known height of the tower, show that the height h of the spire is given by the formula

$$h = \frac{b(a^2 + b^2)}{a^2 - b^2}$$ [10]



[Suggestion: Express tan 2 θ in terms of a and b.]

Part III

Answer at least one question from part III.

25 In a plane triangle, $a = 240$ ft, $b = 264$ ft and $c = 412$ ft. Find C correct to the nearest minute. [10]

26 A lighthouse DC is sighted from two points, A and B, directly west of the lighthouse and on the same horizontal plane with its base C. At A the angle of elevation of D, the top of the lighthouse, is 12° 10′ and at B the angle is 32° 20′. If AB is 1143 feet, find the height of the lighthouse correct to the *nearest foot*. [10]

27 Two forces of 9 lb. and 14 lb. act on a body at an angle of 64° with each other. Find, correct to the *nearest minute*, the angle which the resultant makes with the smaller force. [10]

Part IV

Answer at least one question from part IV.

28 Given right spherical triangle ABC in which C is the right angle; if $a = 25° 6'$ and $b = 137° 19'$, find c correct to the *nearest minute*. [10]

29 The distance between two places, A and B, on the earth's surface is 900 nautical miles. The latitude of A is 40° N and of B 30° N; A is west of B. Find the bearing angle of A from B, that is, the acute angle at B. [10]

Fill in the following lines:

Name of school..... Name of pupil.....

Part I

Answer all questions in part I. Each correct answer will receive 2½ credits. No partial will be allowed. Each answer must be reduced to its simplest form.

1 Express cos 87° as a function of a positive angle less than 45°. 1.....

2 Express cot $(180° + A)$ as a function of A. 2.....

3 Find the value of sin 163° 3.....

4 Find log sin 61° 23′ 4.....

5 Find acute angle A correct to the *nearest minute*, if log cos A = 9.9020 — 10 5.....

6 If A is a positive acute angle and sec $A = 1\frac{7}{8}$, find tan A. 6.....

7 Express in degrees an angle of $\frac{\pi}{3}$ radians. 7.....

8 A circular arc of 30 feet subtends an angle of four radians at the center of its circle. Find the radius of the circle. 8.....

9 Express in degrees an angle of 40 mils. 9.....

10 In which quadrants may the terminal side of an angle lie if its tangent is negative? 10.....

11 Express tan $2x$ in terms of tan x. 11.....

12 If A is an angle in the first quadrant, express tan A in terms of cos A. 12.....

13 Complete the formula sin $(A + B) = \ldots$ 13.....

14 In right spherical triangle ABC, in which C is the right angle, c and b are known. Write the formula that should be used to find B. 14.....

15 Complete the following statement: In the solution of a right spherical triangle ABC, in which C is the right angle, an ambiguous case arises when the given parts are a and .... 15.....

16 Two sides of a parallelogram are 6 and 10 and the included angle is 25°. Find, correct to the *nearest integer*, the area of the parallelogram. 16.....

17 In triangle ABC, $a = 4$, $b = 5$ and $c = 6$. Find the value of cos A. 17.....

18 Find the positive acute angle which satisfies the equation $\tan^2 x - 3 = 0$ 18.....

19 In plane triangle ABC, angle A is acute. If a is less than b and a is greater than $b \sin A$, how many solutions has the triangle? 19.....

20 In spherical triangle ABC, if $a = 125°$, $c = 80°$ and $C = 90°$, in what quadrant is b? 20.....