The University of the State of New York

291st High School Examination

TRIGONOMETRY

Thursday, June 22, 1944 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II, III and IV (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in trigonometry.

The minimum time requirement is five recitations a week for half a school year, or the equivalent.

Answer five questions from parts II, III and IV, including at least one question from each part.

Part II

Answer at least one question from part II.

21 *a* Starting with the formulas for $\sin (A - B)$ and $\cos (A - B)$, derive the formula for $\tan (A - B)$. [5]

b Starting with a formula for $\cos 2A$, derive the formula for $\sin \frac{x}{2}$ in terms of $\cos x$. [5]

22 *a* Express $2 \cos x \csc 2x$ in terms of $\sin x$. [3]

b Solve the equation $3 + 3 \cos x = 2 \sin^2 x$ for all values of x between $0°$ and $360°$. [7]

23 An artillery range spotter is flying at an altitude of h feet. He observes that a gun G and its target T, both in the same horizontal plane, are due west of his position, the target being at the greater distance. The angles of depression of the gun and the target are x and y respectively. Derive a formula for the range r, that is, the distance GT. [10]

24 *a* On the same set of axes, draw the graphs of $y = \sin x$ and $y = 2 \cos x$ as x varies from 0 to 2π radians inclusive at intervals of $\frac{\pi}{6}$ radians. [4, 4]

b Explain how the graphs constructed in answer to *a* enable one to determine the number of solutions between 0 and 2π radians of the equation $\sin x = 2 \cos x$. [2]

Part III

Answer at least one question from part III.

25 In triangle ABC, $a = 328$, $b = 321$ and $c = 205$. Find angle B correct to the *nearest minute*. [10]

26 From a point C at sea level, the angle of elevation of a mountain peak B is 30°. An aviator at A, 4325 feet directly above C, finds that angle BAC is 43°. Find, correct to the *nearest foot*, the height of the mountain peak above sea level. [10]

27 A ship sails 23 miles on a course N 15° E and then 15 miles on a course N 78° E. In what direction, correct to the *nearest minute*, is the ship from the starting point? [10]

Part IV

Answer at least one question from part IV.

28 In spherical triangle ABC, $A = 20° 30'$, $B = 84° 40'$, $c = 90°$. Find C. [10]

29 Find the great circle distance in statute miles between London (Lat. 51° 31' N, Long. 0° 6' W) and Berlin (Lat. 52° 32' N, Long. 13° 24' E). [1 nautical mile = 1.152 statute miles] [10]

Fill in the following lines:

Name of school..... Name of pupil.....

Part I

Answer all questions in this part. Each correct answer will receive 2½ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1 Express in radians an angle of 120°. 1.....

2 Express tan 250° as a function of a positive angle less than 45°. 2.....

3 If sin $A = x$, express sin $(360° - A)$ in terms of x. 3.....

4 If $\cos x = \frac{-\sqrt{2}}{2}$ and $\cot x = -1$, what is the smallest, positive value of x? 4.....

5 Find the positive acute angle which satisfies the equation $2\sin^2 x - 1 = 0$. 5.....

6 What is the maximum value of $\sin 3x$? 6.....

7 Express in mils an angle of 9°. 7.....

8 Express $\tan (x + y)$ in terms of $\tan x$ and $\tan y$. 8.....

9 Express $\tan^2 \frac{x}{2}$ in terms of $\cos x$. 9.....

10 If $\tan x = \frac{1}{2}$, find the value of $\tan 2x$. 10.....

11 Find the logarithm of 0.2347 11.....

12 If $\log \cos x = 9.7611 - 10$ and x is a positive acute angle, find the value of x correct to the *nearest minute*. 12.....

13 In triangle ABC, $a = 5$, $b = 6$, $c = 7$. Find $\cos C$. 13.....

14 If two sides of a triangle are 6 and 8 and the included angle is 30°, find the area of the triangle. 14.....

15 In right spherical triangle ABC, in which C is the right angle, c and A are known. Write the formula that should be used to find b. 15.....

Directions (questions 16–20) — Indicate the correct answer to *each* question by writing the letter *a*, *b*, or *c* on the line at the right.

16 The value of $\tan \frac{5}{6}\pi$ is (*a*) $-\sqrt{3}$, (*b*) $\frac{1}{\sqrt{3}}$, (*c*) $-\frac{1}{\sqrt{3}}$ 16.....

17 As angle A increases from 180° to 270°, the value of $\cos A$ (*a*) decreases from 1 to 0, (*b*) increases from —1 to 0, (*c*) decreases from 0 to —1 17.....

18 The plane triangle in which $a = 11$, $b = 14$, $B = 30°$ has (*a*) two solutions, (*b*) one solution, (*c*) no solution 18.....

19 In spherical triangle ABC, if $a = 144°$, $b = 35°$ and $C = 90°$, then c is (*a*) greater than 90°, (*b*) equal to 90°, (*c*) less than 90° 19.....

20 The polar triangle of an isosceles quadrantal triangle is (*a*) isosceles but not right, (*b*) right but not isosceles, (*c*) both right and isosceles 20.....