The University of the State of New York

221ST HIGH SCHOOL EXAMINATION

TRIGONOMETRY

Thursday, June 19, 1919—1.15 to 4.15 p. m., only

Write at top of first page of answer paper (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in trigonometry.

The minimum time requirement for plane trigonometry is two recitations a week for a school year; for plane and spheric trigonometry three recitations a week for a school year.

Students taking this examination may use textbooks and notes prepared previous to the examination, but there must be no communication among students after the examination has begun.

Candidates for plane trigonometry should answer six questions, including three from group I and three from group II.

Candidates for plane and spheric trigonometry should answer six questions, including two from group I, two from group II and two from group III.

Candidates for spheric trigonometry who have previously passed plane trigonometry should answer three questions from group III.

A, B and C represent the angles of a triangle ABC; a, b and c represent the respective opposite sides. In a right triangle, C represents the right angle.

Give special attention to neatness and arrangement of work.

Group I

1 *a* Explain why log cos A can not exceed 0. For what other function of A is this true?

b For purposes of computation why is it an advantage to have trigonometric formulas expressed as products or quotients rather than as sums or differences?

c When the base is 10, what are the logarithms of the following: 1; 10; 10000; .1; 0.01; 0.0001?

When the base is 2, what are the logarithms of 1, 2, 4 and 256?

2 *a* When $\sin A = -\frac{1}{2}\sqrt{2}$, write the values of the cosine, tangent and cotangent for the quadrant in which $\tan A$ is negative.

b Name the four smallest positive angles for which $\sin A = -\frac{1}{2}\sqrt{2}$

3 *a* Simplify $\frac{\sin 75^\circ - \sin 15^\circ}{\sin 75^\circ + \sin 15^\circ}$ without making use of the tables for natural functions.

b Express the following as functions of positive angles less than 45°: cot 142.5°; $-\cos(-35.75^\circ)$

4 Prove the following:

a $\frac{\cos A}{1-\tan A} + \frac{\sin A}{1-\cot A} = \sin A + \cos A$

b $\left(\cos\frac{A}{2} - \sin\frac{A}{2}\right)^2 = 1 - \sin A$

Group II

5 An observer in a war balloon observes the angle of depression of an enemy battery to be 27.5°; an instrument registers the height as 3250 feet. At what distance from a point on the ground directly below the observer is the battery located, if the point and the battery are on the same horizontal plane?

6 An observer in a war balloon at a definite height locates two distant forts; if the forts and the point on the ground directly below the observer are on the same horizontal plane, what further observations should be made and how should these observations be made use of to determine the distance between the forts?

7 From two points on the same level 875 feet apart, on opposite sides of a tower and directly in line with it, the angles of elevation are observed to be 29° 30′ and 21° 15′; find the height of the tower.

8 Given $a=34.16$, $b=26$, $c=16.7$; determine the size of the largest angle.

Group III

9 In a right spheric triangle, $a=117^\circ\ 10'$, $B=126^\circ\ 52'$; solve the triangle. Check your solution.

10 Are the following spheric triangles possible? If impossible, state what conditions are violated:

(1) $a=189^\circ$ $b=100^\circ$ $c=70^\circ$
(2) $a=140^\circ$ $b=150^\circ$ $c=160^\circ$
(3) $A=100^\circ$ $B=200^\circ$ $C=300^\circ$
(4) $A=200^\circ$ $B=235^\circ$ $C=100^\circ$

11 In the spheric triangle, given $a=65^\circ\ 20'$, $b=55^\circ\ 18'$, $c=96^\circ$; find C.

12 New York lies in latitude 40°45′N., longitude 73°58′W.; Brest lies in latitude 43° 23′ N., longitude 4° 29′ W. Find the shortest distance in miles between these places. [Radius of earth = 3956 miles]