The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

TRIGONOMETRY

Monday, January 27, 1964 — 1:15 to 4:15 p.m., only

Name of pupil..... Name of school.....

Name of teacher.....

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form.

1 Express tan $(-100°)$ as a function of a positive acute angle. 1.....

2 Find the number of degrees in $\frac{2}{3}\pi$ radians. 2.....

3 If $\tan^2 \theta = 64$, find the values of cot θ. 3.....

4 Angle A terminates in quadrant III and tan $A = \frac{3}{4}$. Express cos $2A$ as a fraction. 4.....

5 Find the number of degrees in the smallest positive angle θ for which $\tan^2 \theta - 2 \tan \theta + 1 = 0$. 5.....

6 Find the value of log cot 54° 21′. 6.....

7 Find to the *nearest minute* the positive acute angle θ if tan $\theta = 0.8312$. 7.....

8 If angle A terminates in quadrant IV, express sin A in terms of cos A. 8.....

9 What is the period of $y = \frac{1}{2} \cos 2x$? 9.....

10 What is the maximum value of $2 \cos \frac{1}{2}x$? 10.....

11 Find the number of radians in the central angle which intercepts an arc of 0.5 inch on a circle whose radius is 2 inches. 11.....

12 If A is a positive angle which terminates in quadrant II and cos $A = -\frac{1}{8}$, find the value of sin $\frac{1}{2}A$. 12.....

13 In triangle ABC, $a = 2\sqrt{3}$, $b = 3\sqrt{3}$ and sin $A = 0.4$. Find the value of sin B. 13.....

[OVER]

191

14 In triangle ABC, $a = 3$, $b = 4$ and $C = 45°$. What is the area of triangle ABC? 14.....

15 In triangle ABC, $a = 4$, $b = \sqrt{5}$ and $c = 3$. Find the value of $\cos B$. 15.....

16 Express $\sec (\pi + \theta)$ in terms of $\sec \theta$. 16.....

17 How many distinct noncongruent triangles, if any, can be constructed using the data $A = 60°$, $a = 9$ and $b = 10$? 17.....

18 A is 5 miles due north of C, and B is 5 miles due west of C. Express the course or bearing to be followed in traveling from A to B. 18.....

19 In triangle ABC, $a = 101$, $b = 99$ and $C = 10°$. Find the value of $\tan \frac{1}{2}(A - B)$ to the *nearest hundredth*. 19.....

20 From the top of a tower 40 feet directly above a point A on level ground, the angle of depression of point B on the ground is observed to be $8° 30'$. Find the distance from A to B to the *nearest foot*. 20.....

21 Find the positive value of $\tan (2 \arcsin \frac{1}{2})$. 21.....

Directions (22–28): Write on the line at the right of *each* of the following the *number* preceding the expression that best completes the statement or answers the question.

22 Which of the following expressions is true for any scalene triangle ABC?

(1) $2bc \cos A = b^2 + c^2 + a^2$

(2) $c \sin A = a$

(3) $\dfrac{a - b}{a + b} = \dfrac{\tan (A - B)}{\tan (A + B)}$

(4) $a \sin B = b \sin A$

22.....

23 Sin x and cos x both increase throughout the interval as x increases from

(1) $0°$ to $90°$

(2) $90°$ to $180°$

(3) $180°$ to $270°$

(4) $270°$ to $360°$

23.....

24 If θ is an acute angle less than $45°$, then $\tan 2\theta(1 - \tan \theta)$ is equivalent to

(1) $\tan 2\theta - \tan 3\theta$

(2) 2

(3) $\dfrac{2 \tan \theta}{1 + \tan \theta}$

(4) $\dfrac{2 \tan \theta}{1 - \tan \theta}$

24.....

25 Using the fact that $\cos 75° = \cos (45° + 30°)$, it may be shown that $\cos 75°$ equals

(1) $\dfrac{\sqrt{2} - \sqrt{6}}{4}$

(2) $\dfrac{\sqrt{6} - \sqrt{2}}{4}$

(3) $\dfrac{\sqrt{2} + \sqrt{6}}{4}$

(4) $\dfrac{-\sqrt{2} - \sqrt{6}}{4}$

25.....

162

26 The expression $\cos 5\theta + \cos \theta$ is equivalent to

(1) $2 \sin 3\theta \cos 2\theta$
(2) $2 \cos 3\theta \sin 2\theta$
(3) $2 \cos 3\theta \cos 2\theta$
(4) $-2 \sin 3\theta \sin 2\theta$

26.....

27 Log csc A is equivalent to

(1) $-\log \sin A$
(2) $\dfrac{1}{\log \sin A}$
(3) $1 - \log \sin A$
(4) $\log (1 - \sin A)$

27.....

28 A value of x between 0 and π for which the expression $\dfrac{1 - \sin 2x}{\cos 2x}$ is *not* defined is

(1) $\dfrac{\pi}{6}$
(2) $\dfrac{\pi}{4}$
(3) $\dfrac{\pi}{3}$
(4) $\dfrac{\pi}{2}$

28.....

Directions (29–30): Indicate whether the following statements are true for
(1) all real values of θ,
(2) some, but not all, real values of θ,
(3) no real value of θ,
by writing on the line at the right the number 1, 2 or 3.

29 $\sin 2\theta = 2 \sin \theta$ 29.....

30 $\sin \theta > -\sin \theta$ 30.....

Part II

Answer four questions from this part. Show all work unless otherwise directed.

31 Solve the following equation for all values of A between $0°$ and $360°$. [Express the values of A to the *nearest degree*.] [10]

$$6 \sin^2 A - \cos A - 4 = 0$$

32 *a* On the same set of axes, *sketch* the graphs of $y = \sin 2x$ and $y = \frac{1}{2} \cos x$ as x varies from 0 to 2π radians. [Label each curve with its equation.] [4, 4]

b From the graphs made in answer to part *a*, determine the number of solutions of $\sin 2x = \frac{1}{2} \cos x$ as x varies from 0 to 2π radians. [2]

33 *a* Beginning with a formula for $\cos 2\theta$, *derive* a formula for $\sin \frac{1}{2}x$ in terms of $\cos x$. [5]

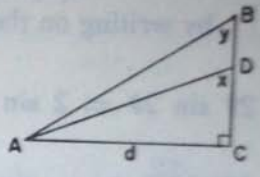
b Prove that the following equality is true for all values of x for which the members are defined: [In other words, prove the equality is an identity.] [5]

$$\cot x = \frac{\sin (x + y)}{\sin x \sin y} - \cot y$$

34 Coast Guard Station B is 20 miles northeast (N 45° E) of Station A. A distress signal is received from a ship along a bearing of 150° (S 30° E) from Station A and 215° (S 35° W) from Station B. Find to the *nearest mile* the distance between the ship and the closer station. [6, 4]

35 Given right triangle ABC, hypotenuse AB, D any point on BC and line AD drawn. Let $AC = d$, $\angle ADC = x$ and $\angle ABC = y$.

Prove: $BD = \dfrac{d \sin (x - y)}{\sin x \sin y}$ [10]



36 Answer *either* *a* or *b* but *not* both: [10]

a In triangle ABC, $a = 340$, $b = 440$ and $c = 480$. Find to the *nearest ten minutes* the measure of angle B.

OR

b In triangle ABC, $a = 138$, $b = 158$ and $C = 111°$. Find to the *nearest ten minutes* the measure of angle A.

FOR TEACHERS ONLY

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SCORING KEY

TRIGONOMETRY

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Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 22–30, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3 or 4.

(1) tan 80° *or* cot 10°

(2) 120

(3) $\pm\frac{1}{8}$

(4) $\frac{7}{25}$

(5) 45

(6) 9.8556 — 10

(7) 39° 44′

(8) $-\sqrt{1-\cos^2 A}$

(9) π *or* 180°

(10) 2

(11) $\frac{1}{4}$

(12) $\frac{3}{4}$

(13) 0.6

(14) $3\sqrt{2}$ *or* 4.2

(15) $\frac{5}{6}$

(16) $-\sec\theta$

(17) 2

(18) S 45° W *or* 225°

(19) 0.11

(20) 268

(21) $\sqrt{3}$ *or* 1.7

(22) 4

(23) 4

(24) 3

(25) 2

(26) 3

(27) 1

(28) 2

(29) 2

(30) 2

[OVER]

Part II

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics*. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(31) $60°, 132°, 228°, 300°$ [10]

(32) *b* four [2]

(34) Analysis [6]
4 [4]

(36) *a* $62° 10'$ [10]
b $31° 50'$ [10]