January 24, 1963

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form.

1. Find the numerical value of $\tan \frac{\pi}{3}$. 1_____

2. If B is a positive acute angle and $\cos 30° = \frac{1}{\sec B}$, find the number of degrees in angle B. 2_____

3. Express 4.5π radians in degrees. 3_____

4. If $\sin A = \frac{2}{3}$ and $\sec A$ is negative, evaluate $\tan A$. 4_____

5. Find $\log \tan 27° 34'$. 5_____

6. Find to the *nearest minute* the smallest positive angle whose cosine is 0.9525. 6_____

7. If x is a positive acute angle, express $\tan x$ in terms of $\csc x$. 7_____

8. Evaluate $\cos (270° - A)$, if $\sin A = \frac{1}{3}$ and angle A is acute. 8_____

9. In a circle a central angle intercepts an arc equal in length to the diameter of the circle. How many radians are there in this central angle? 9_____

10. If $2 \sin 3x \cos 3x = 1$ and x is a positive angle, find in degrees the smallest value of x. 10_____

11. Express $\tan \frac{\theta}{2}$ in terms of $\cos \theta$. 11_____

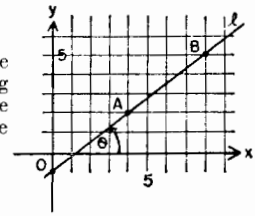
12. Express in degrees the value of x between 180° and 360° which satisfies the equation $\sin x = \csc x$. 12_____

13. If $\tan (x + y) = 4$ and $\tan y = 2$, find the numerical value of $\tan x$. 13_____

14. If $4x$ is a positive acute angle and $\sin 4x = \cos x$, find the number of degrees in x. 14_____

15. If y is a positive angle less than 180°, find in degrees the value of y for which $y = \arctan 1$. 15_____

16. Points A (4, 2) and B (8, 5) lie on line l as shown in the accompanying figure. Find the tangent of the acute angle θ which line l makes with the positive x-axis.



16_____

17. The sides of a triangle are 6, 10 and 14. Compute the value of the cosine of the smallest angle of the triangle. 17_____

18. Find the area of triangle ABC in which $AB = 20$, $BC = 10$ and angle $B = 45°$. 18_____

19. Express cos 295° as a function of a positive acute angle. 19_____

20. In triangle ABC, $a = 10$, $b = 8$ and $\sin B = \frac{3}{4}$. Find $\sin A$. 20_____

21. If x and y are acute angles, and if $\sin x = \frac{12}{13}$ and $\sin y = \frac{3}{5}$, find the value of $\sin (x + y)$. 21_____

22. Solve for x: $\log (x - 1) + \log (x + 3) = \log (x^2)$ 22_____

23. What is the period of the graph which represents the function $y = 3 \cos \frac{1}{2}x$? 23_____

24. The base of an isosceles triangle is 10 and the altitude to the base is 12. Find the sine of a base angle. 24_____

25. In triangle ABC, $a = 6$, $c = 4$ and angle $B = 76°$. Find to the *nearest hundredth* the numerical value of $\tan \left\{ \frac{A - C}{2} \right\}$. 25_____

Directions (26-30): Indicate the correct completion for *each* of the following by writing on the line at the right the number 1, 2, 3 or 4.

26. The expression $\sin x - \sin y$ is equivalent to (1) $\sin (x - y)$ (2) $2 \cos \frac{x + y}{2} \sin \frac{x - y}{2}$ (3) $2 \sin \frac{x + y}{2} \cos \frac{x - y}{2}$ (4) $\sin x \cos y - \cos x \sin y$ 26_____

27. The statement $\sin x + \cos x = 1$ is true for (1) only one value of x (2) more than one value of x, but not all values (3) all values of x (4) no values of x 27_____

28. If the data $A = 60°$, $a = 15$ and $b = 20$ are used, (1) triangle ABC must be acute (2) triangle ABC must be obtuse (3) triangle ABC may be either acute or obtuse (4) no triangle can be constructed 28_____

29. The expression $\sin 3\theta \cos \theta - \cos 3\theta \sin \theta$ is equivalent to (1) $\cos \theta$ (2) $\sin \theta$ (3) $\sin 2\theta$ (4) $\cos^2 \theta - \sin^2 \theta$ 29._____

30. As angle A increases from 45° to 225°, $\sin A$ (1) increases throughout the interval (2) increases, then decreases (3) decreases throughout the interval (4) decreases, then increases 30._____

Part II

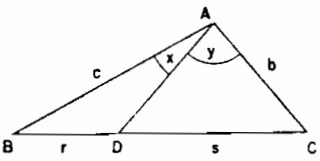
Answer four questions from this part. Show all work unless otherwise directed.

31. *a* Find all nonnegative values of θ less than 360° which satisfy the equation $\sin 2\theta + \cos (90° - \theta) = 0$. [8]

b What value of θ is greater than 360° and less than 540° satisfies the equation in part *a*? [2]

32. *a* Starting with the formula for $\cos (x + y)$, derive a formula for $\cos 2x$ in terms of $\cos x$. [4]

b In the accompanying figure, ABC is a triangle and AD is a line meeting BC in D. If $BD = r$, $DC = s$, $AB = c$, $AC = b$, angle $BAD = x$ and angle $DAC = y$, show that $\dfrac{\sin x}{\sin y} = \dfrac{br}{cs}$. [6]



33. *a* *Sketch* and *label* the graph of $y = \frac{1}{2} \sin 2x$ as x varies from $-\dfrac{\pi}{2}$ radians to $+\dfrac{3\pi}{2}$ radians, inclusive. [6]

b On the same set of axes, *sketch* and *label* the graph of $y = \cos x$ as x varies from $-\dfrac{\pi}{2}$ radians to $+\dfrac{3\pi}{2}$ radians, inclusive. [3]

c How many values of x between 0 and π radians satisfy the equation $\frac{1}{2} \sin 2x = \cos x$? [1]

34. *a* Prove that the following equality is true for all values of x for which the members are defined: (In other words, prove the equality is an identity.) [8]

$$\frac{1}{\cos 2x} = \frac{2}{1 - \tan^2 x} - 1$$

b Division by zero is undefined. Determine *two* positive values of x less than 180° for which $\dfrac{1}{\cos 2x}$, the left member of the equality in part *a*, is undefined. [2]

35. *a* In triangle ABC, angle B, angle C and side a are given. *Derive* a formula for h, the altitude drawn to side a, in terms of the given parts B, C and a. [6]

b Find to the *nearest tenth* the altitude to side a in triangle ABC if angle $B = 30°$, angle $C = 46°$ and $a = 10$. [4]

36. Answer *either* *a* or *b*:

a Two forces of 720 pounds and 640 pounds, respectively, act upon a body at an angle of 42° 40′ with each other. Find to the *nearest ten minutes* the angle that the resultant force makes with the smaller force. [10]

OR

b Acute triangle ABC is inscribed in a circle. Angle B intercepts an arc of 142°, $AB = 17.5$ and $AC = 23.8$.

(1) Find to the *nearest degree* the three angles of triangle ABC. [8]

(2) Name the longest side of triangle ABC. [2]