

TRIGONOMETRY

Monday, January 25, 1960 — 1:15 to 4:15 p.m., only

Name of pupil..... Name of school.....

Part I

Answer all questions in this part. Each correct answer will receive $2\frac{1}{2}$ credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form.

1 Express $\tan (-310^\circ)$ as a function of a positive acute angle. 1.....2 In a circle, a central angle of 2.5 radians intercepts an arc of 15 inches.
Find the number of inches in the radius of the circle. 2.....

3 Express in degrees an angle of 2 radians. 3.....

4 If A is a positive acute angle, express $\cot A$ in terms of $\sec A$. 4.....5 In $\triangle ABC$, $A = 30^\circ$, $C = 105^\circ$ and $b = 6$. Find a . 5.....6 In $\triangle ABC$, $A = 60^\circ$, $b = 5$ and $c = 8$. Find a . 6.....7 Find the positive value of $\sin (\arctan \frac{1}{3})$. 7.....8 Find $\log \tan 37^\circ 17'$. 8.....9 Find to the nearest minute the positive acute angle A if $\cos A = 0.7720$. 9.....10 If $\tan A = \frac{1}{2}$, find $\tan 2A$. 10.....

11 Express $\cos 82^\circ + \cos 32^\circ$ as a product of two functions.

11.....

12 As angle θ increases from 180° to 360° , $\sin \theta$ (1) increases throughout the interval (2) decreases throughout the interval (3) decreases, then increases (4) increases, then decreases
Which is correct: 1, 2, 3 or 4?

12.....

13 Find the value of the acute angle θ for which the following is true:
 $2 \cos^2 \theta - \sqrt{3} \cos \theta = 0$

13.....

14 What is the minimum value of $\cos 3x$?

14.....

15 How many triangles can be constructed using the data $A = 95^\circ$,
 $b = 9$ and $a = 8$?

15.....

16 In parallelogram $ABCD$, $AB = 10$, $AD = 6$ and angle $A = 44^\circ 40'$.
Find to the nearest integer the area of the parallelogram.

16.....

Directions (17–20): Indicate whether the following statements are true for

a all real values of x

b some but not all real values of x

c no real value of x

by writing on the line at the right the letter *a*, *b* or *c*.

17 $\sin^2 x = 2 - \cos^2 x$

17.....

18 $\sin(90^\circ + x) = \cos x$

18.....

19 $\sin x + \cos x = 1$

19.....

20 $\cos 6x = 2 \cos^2 3x - 1$

20.....

Part II

Answer three questions from this part. Show all work unless otherwise directed.

- 21 Find all values of x between 0° and 360° which satisfy the equation $2 \sin x + 4 \cos 2x = 3$.
[Express approximate values of x to the nearest degree.] [10]

- 22 a On the same set of axes, sketch the graphs of $y = 2 \sin \frac{1}{2}x$ and $y = \frac{1}{2} \cos x$ as x varies from 0 to 2π radians. [4, 4]

- b From the graphs made in answer to part a, determine the quadrants in which x lies if $2 \sin \frac{1}{2}x = \frac{1}{2} \cos x$. [2]

- 23 a Starting with a formula for $\cos 2A$, derive the formula for $\cos \frac{x}{2}$ in terms of $\cos x$. [5]

- b Starting with the formulas for $\sin (x - y)$ and $\cos (x - y)$, derive the formula for $\tan (x - y)$ in terms of $\tan x$ and $\tan y$. [5]

- 24 a Prove that the following equality is an identity: [5]

$$\tan x = \frac{\sin 2x}{1 + \cos 2x}$$

- b Show that $\sin (45^\circ + x) + \sin (45^\circ - x)$ may be reduced to $\sqrt{2} \cos x$. [5]

- 25 Given acute triangle ABC . Show that $\tan B = \frac{b \sin A}{c - b \cos A}$.
[Hint: Draw the altitude from C .] [10]

Part III

Answer two questions from this part. Show all work.

- 26 In $\triangle ABC$, $a = 37.6$, $b = 26.4$ and $C = 70^\circ 20'$. Find A to the nearest ten minutes. [10]

- 27 Two forces acting upon a body make an angle of $103^\circ 30'$ with each other. The magnitude of the first force is 386 pounds. If the resultant makes an angle of $47^\circ 10'$ with the first force, what is the magnitude of the resultant, to the nearest pound? [10]

- 28 Two ships leave point A at 10:30 a.m. One travels in a direction of 049° (N 49° E) at 12 miles per hour and the other travels in a direction of 135° (S 45° E) at 14 miles per hour. How far apart, to the nearest mile, will they be at noon? [5, 5]

- 29 In $\triangle ABC$, $AB = 28.7$, $BC = 36.4$ and $CA = 14.3$. Find B to the nearest ten minutes. [10]

FOR TEACHERS ONLY

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INSTRUCTIONS FOR RATING TRIGONOMETRY

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Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use check marks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow $2\frac{1}{2}$ credits for each correct answer; allow no partial credit. For questions 17–20, allow credit if the pupil has written the correct answer instead of the letter *a*, *b* or *c*.

(1) $\tan 50^\circ$ or $\cot 40^\circ$

(2) 6

(3) $\left(\frac{360}{\pi}\right)^\circ$ or 114.6° or 115°

(4) $\frac{1}{\sqrt{\sec^2 A - 1}}$

(5) $3\sqrt{2}$

(6) 7

(7) $\frac{1}{\sqrt{10}}$ or $\frac{\sqrt{10}}{10}$

(8) $9.8816 - 10$

(9) $39^\circ 28'$

(10) $\frac{4}{3}$

(11) $2 \cos 42^\circ \cos 10^\circ$

(12) 3

(13) 30°

(14) -1

(15) none

(16) 42

(17) *c*

(18) *a*

(19) *b*

(20) *a*

[OVER]

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics*. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

Part II

$$(21) 30^\circ, 150^\circ, 194^\circ, 346^\circ \quad [10]$$

$$(22) b \text{ I and IV} \quad [2]$$

Part III

$$(26) 68^\circ 50' \quad [10]$$

$$(27) 451 \quad [10]$$

$$(28) \text{ Analysis} \quad [5]$$

$$27 \text{ miles} \quad [5]$$

$$(29) 21^\circ 30' \quad [10]$$