TRIGONOMETRY

Thursday, January 22, 1959 — 9:15 a.m. to 12:15 p.m., only

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Name of pupil..Name of school..

Part I

Answer all questions in this part. Each correct answer will receive 2½ credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form.

1 Express sec 140° as a function of a positive acute angle.   1.....................

2 Find the value of cot $\dfrac{5\pi}{4}$.   2.....................

3 Find the number of degrees in the smallest positive angle x if $\cos x = \dfrac{\sqrt{2}}{2}$ and tan x is negative.   3.....................

4 Express in terms of a the positive value of cos (arc sin a).   4.....................

5 Find sin 57° 24′.   5.....................

6 Find, to the *nearest minute*, the positive acute angle A, if log tan $A = 9.7820-10$.   6.....................

7 In triangle ABC, $a = 5$, $A = 30°$, $C = 45°$. Find side c.   7.....................

8 In triangle ABC, $a = 5$, $b = 6$. Express cos C in terms of side c.   8.....................

9 Express sin 70° — sin 40° as a product of two functions.   9.....................

10 If x is in the first quadrant and $\cos x = \dfrac{7}{25}$, find the value of cos $\frac{1}{2}x$.   10.....................

11 Two forces of 5 pounds and 12 pounds act upon a body at an angle of 90° with each other. Find in pounds the magnitude of the resultant.   11.....................

12 How many triangles can be constructed using the data $A = 30°$, $b = 12$ and $a = 5$?   12.....................

[1]

Directions (13–16) : Indicate the correct completion for *each* of the following by writing on the line at the right the letter *a, b, c* or *d.*

13 If, in triangle ABC, $a = 2b$, then $\dfrac{\tan \frac{1}{2}(A + B)}{\tan \frac{1}{2}(A - B)}$ is equal to

    (*a*) 6   (*b*) 2   (*c*) 3   (*d*) 4

13. . . . . . .

14 The maximum value of $2 \sin x \cos x$ is   (*a*) 1   (*b*) 2   (*c*) $\frac{1}{2}$ (*d*) 4

14. . . . . . .

15 If x is an acute angle, $\log \tan x$ is equal to   (*a*) $\dfrac{\log \sin x}{\log \cos x}$ (*b*) $\dfrac{\log \cos x}{\log \sin x}$

    (*c*) $\log \sin x - \log \cos x$ (*d*) $\log \cos x - \log \sin x$

15. . . . . . .

16 In a circle whose radius is r inches, a central angle of two radians intercepts an arc of   (*a*) 2 inches   (*b*) $2r$ inches   (*c*) $2r^2$ inches   (*d*) $2\pi r$ inches

16. . . . . . .

Directions (17–20) : Indicate whether the following statements are true for (*a*) all real values of x, (*b*) some but not all real values of x or (*c*) no real value of x, by writing on the line at the right the letter *a, b* or *c.*

17 $\sin^2 \dfrac{x}{2} + \cos^2 \dfrac{x}{2} = \frac{1}{2}.$

17. . . . . . .

18 $\sec x = \frac{1}{2} \sin x.$

18. . . . . . .

19 $\cos 2x = \cos^2 x - \sin^2 x.$

19. . . . . . .

20 $\sin x - \cos x = 0.$

20. . . . . . .

[2]

Part II

Answer three questions from this part. Show all work unless otherwise directed.

21 Find all positive values of x less than $360°$ which satisfy the equation
$\cos 2x - 3 \cos x - 1 = 0$. [10]

22 *a* On the same set of axes, *sketch* the graphs of $y = \cos 2x$ and $y = 2 \cos x$, as x varies from 0 to 2π radians. [4, 4]

b From the graphs made in answer to part *a*, determine the maximum value of $\cos 2x - 2 \cos x$. [2]

23 *a* Prove the identity: $\tan x + \cot x = \sec x \csc x$ [5]

b In triangle ABC, $B = 45°$. Using the fact that $C = 180° - (A + B)$, show that
$$\sin C = \frac{\sqrt{2}}{2} (\sin A + \cos A). \quad [5]$$

24 *a* Starting with the formula for $\tan (x + y)$, derive

(1) the formula for $\tan (x - y)$ by substituting $(-y)$ for y [2]

(2) the formula for $\tan 2x$ [2]

b Using the formula for $\tan (x - y)$, in which $x = 45°$ and $y = 30°$, express $\tan 15°$ in radical form. [3]

c Using the formula for $\tan 2x$, and given that $\tan 30° = \frac{\sqrt{3}}{3}$, show that $\tan 60° = \sqrt{3}$. [3]

25 A surveyor wishes to determine the height CD of a vertical building. CAB is a horizontal straight line. The angle of elevation of D when measured from A is x degrees, and when measured from B is y degrees. The distance between A and B is d feet. Show that $CD = \dfrac{d \sin x \sin y}{\sin (x - y)}$. [10]
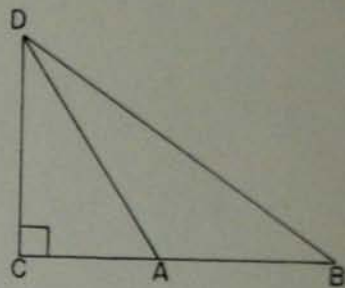
Part III

Answer two questions from this part. Show all work.

26 Two angles of a triangle are $46° 40'$ and $58° 30'$, and the included side is 64.3 feet. Find, to the *nearest tenth of a foot*, the length of the shortest side of the triangle. [10]

27 In triangle ABC, $a = 12$, $b = 18$, $c = 20$.

a Find B to the *nearest degree*. [5]

b Using the result of part *a*, find the area of triangle ABC to the *nearest square unit*. [5]

28 Two forces act on an object. The first force has a magnitude of 86 pounds and makes an angle of $34°$ with the resultant. The magnitude of the resultant is 128 pounds. Find, to the *nearest degree*, the angle that the second force makes with the resultant. [10]

29 Point B is N $70°$ E of A. Point C is N $42°$ W of B. The distance from C to A is 26 miles and the distance from C to B is 18 miles. Find, to the *nearest degree*, the direction of C from A. [5, 4, 1]

[3]

FOR TEACHERS ONLY

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INSTRUCTIONS FOR RATING
TRIGONOMETRY

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Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use check marks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow $2\frac{1}{2}$ credits for each correct answer; allow no partial credit. Do not allow credit if the answer to question 5 is not expressed to four decimal places. For questions 13–20, allow credit if the pupil has written the correct answer instead of the letter *a, b* or *c*.

(1) $- \sec 40°$ *or* $- \csc 50°$

(2) 1

(3) 315

(4) $\sqrt{1-a^2}$

(5) 0.8424

(6) 31° 11′

(7) $5\sqrt{2}$

(8) $\dfrac{61 - c^2}{60}$

(9) $2 \cos 55° \sin 15°$

(10) $\frac{4}{8}$

(11) 13

(12) none

(13) *c*

(14) *a*

(15) *c*

(16) *b*

(17) *c*

(18) *c*

(19) *a*

(20) *b*