The University of the State of New York

326TH HIGH SCHOOL EXAMINATION

TRIGONOMETRY

Thursday, January 26, 1956 — 9.15 a.m. to 12.15 p.m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in trigonometry.

The minimum time requirement is four or five recitations a week for half a school year.

Part II

Answer three questions from this part. Show all work unless otherwise directed.

21 Find all values of x between 0° and 360° which satisfy the equation $5 \sin^2 x - 7 \cos x + 1 = 0$. [Answers may be expressed to the *nearest degree.*] [10]

22 *a* Starting with the formulas for sin $(A + B)$ and cos $(A + B)$, *derive* the formula for tan $(A + B)$ in terms of tan A and tan B. [6]

b *Prove* the following identity: [4]

$$\frac{\cos A + \sin A \tan A}{\sin A \sec A} = \csc A$$

23 *a* *Sketch* the graph of $y = \tan x$ as x varies from 0 to 2π radians. [4]

b On the set of axes used in part *a, sketch* the graph of $y = 2 \cos x$ as x varies from 0 to 2π radians. [4]

c From the graphs made in answer to parts *a* and *b,* determine a value of x for which $\tan x - 2 \cos x = 2$. [2]

24 In the case of each of the following, the statement is *always* true, *sometimes* true or *never* true. List the letters *a–e* on your answer paper and after *each* letter indicate the correct answer by writing the word *always, sometimes* or *never.* [10]

a If $\cos x$ is negative, $\tan x$ is negative.

b Sin $(-x)$ is equal to $-\sin x$.

c If $\sin 2x = a$ and $\cos 2x = b$, then $a^2 + b^2$ is equal to 1.

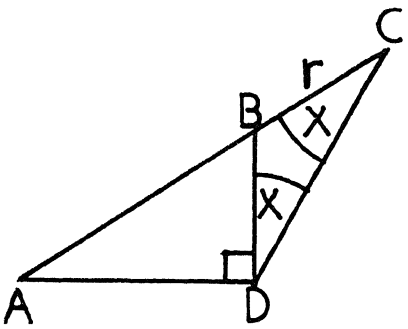
d If x is an angle between 90° and 270°, $\sin \frac{1}{2}x$ is negative.

e If two sides and the angle opposite one of these sides is given, a triangle is determined.

25 In triangle ADC, B is a point on AC such that BD is perpendicular to AD, angles BDC and BCD are each represented by x and BC is represented by r. Show that *each* of the following relationships is true:

a $DC = 2r \cos x$ [4]

b $AC = \dfrac{2r \cos^2 x}{\cos 2x}$ [6]



[OVER]

Part III

Answer two questions from this part. Show all work.

26 In triangle ABC, $a = 57.9$, $b = 34.4$ and angle $C = 114° 20'$. Find angle A to the ***nearest ten minutes***. [10]

27 From a ship K, radio stations A and B bear N 48° E and S 26° E respectively. Station A is known to be 125 miles from B and in a direction N 17° E from B. Find, to the ***nearest mile***, the distance of the ship from A. [5, 5]

28 A body is acted on by a force of 436 pounds and by a second force of 322 pounds. The resultant force is 594 pounds. Find, to the ***nearest degree***, the angle formed by the resultant and the greater force. [10]

29 A vertical pole and a tower 136 feet high stand on the same horizontal plane. From the top of the tower the angles of depression of the top and bottom of the pole are 32° 20′ and 48° 20′ respectively. Find, to the ***nearest foot***, the height of the pole. [4, 6]

Fill in the following lines:

Name of pupil.... .....Name of school.....

Part I

Answer all questions in this part. Each correct answer will receive 2½ credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form.

1 Express sin 230° as a function of a positive acute angle. 1.....

2 Express in radians an angle of 99°. 2.....

3 Find the number of radians in a central angle whose sides intercept an arc on a circle equal in length to the diameter of the circle. 3.....

4 If $\theta = \cos^{-1} \frac{\sqrt{3}}{2}$, find the positive acute angle θ. 4.....

5 Find the smallest positive value of x that satisfies the equation $\tan^2 x - 3 = 0$. 5.....

6 Find n if $\log n = 3.2912$ 6.....

7 Find cos 41° 33′. 7.....

8 Find to the *nearest minute* the positive acute angle A if $\log \sin A = 9.9206 - 10$. 8.....

9 One leg of a trapezoid is 10 and makes an angle of 53° 10′ with the longer base. Find the altitude of the trapezoid. [Answer may be expressed to the *nearest integer*.] 9.....

10 In triangle ABC, $A = 30°$, $B = 45°$ and $b = 20\sqrt{2}$. Find a. 10.....

11 In triangle ABC, $a = 5$, $b = 7$ and $\cos C = \frac{1}{7}$. Find c. 11.....

12 Two sides of a parallelogram are a and b and the angle between them is C. Express the area of the parallelogram in terms of a, b and angle C. 12.....

13 If $\cos x = \frac{7}{9}$, find $\cos 2x$. 13.....

14 If A is a positive acute angle and $\sec A = r$, express $\tan A$ in terms of r. 14.....

15 If the maximum value of $r \sin 3x$ is 6, find r. 15.....

[OVER]

Directions (16–20): Indicate the correct completion for *each* of the following by writing on the line at the right the letter *a*, *b* or *c*.

16 If $\cos x = \frac{7}{11}$ and x is acute, $\sin \frac{x}{2}$ is equal to (*a*) $\sqrt{\frac{2}{11}}$ (*b*) $\sqrt{\frac{8}{11}}$ (*c*) $\sqrt{\frac{9}{11}}$ 16.....

17 Sin $3x$ + sin x is equal to (*a*) sin $4x$ (*b*) 2 sin $2x$ cos x (*c*) 2 sin x cos $2x$ 17.....

18 The statement $\sin^2 x - \cos^2 x = 1$ is (*a*) true for all values of x (*b*) true for only certain values of x (*c*) not true for any values of x 18.....

19 Sec $\theta = \infty$ (infinity) when θ is equal to (*a*) 0° (*b*) 90° (*c*) 180° 19.....

20 As x increases from 0 to 2π radians, the graph of $y = \cos \frac{1}{2}x$ intersects the x-axis in (*a*) one point (*b*) two points (*c*) four points 20.....

FOR TEACHERS ONLY

INSTRUCTIONS FOR RATING
TRIGONOMETRY

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Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use check marks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow $2\frac{1}{2}$ credits for each correct answer; allow no partial credit. Do not allow credit if the answer to question 7 is not expressed to four decimal places. For questions 16–20, allow credit if the pupil has written the correct answer instead of the letter *a*, *b* or *c*.

(1) $-\cos 40°$ *or* $-\sin 50°$
(2) $\frac{11\pi}{20}$
(3) 2
(4) 30°
(5) 60°
(6) 1955
(7) 0.7484
(8) 56° 24′
(9) 8 *or* 8.004
(10) 20
(11) 8
(12) $ab \sin C$
(13) $\frac{17}{81}$
(14) $\sqrt{r^2 - 1}$
(15) 6
(16) *a*
(17) *b*
(18) *b*
(19) *b*
(20) *a*