The University of the State of New York

317TH HIGH SCHOOL EXAMINATION

TRIGONOMETRY

Thursday, January 22, 1953 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in trigonometry.

The minimum time requirement is four or five recitations a week for half a school year, or the equivalent.

Answer five questions from parts II and III, including at least two questions from each part.

Part II

Answer at least two questions from part II.

21 *a* Derive the Law of Sines. [Consider only the case in which the triangle is acute.] [6]

b Prove the identity: $\frac{2 \tan x}{1 + \tan^2 x} = \sin 2x$ [4]

22 Find all values of θ between 0° and 360° that satisfy the equation $4 \cos^2 \theta - 3 \sin \theta = 3$. [Express approximate values of θ correct to the *nearest degree.*] [10]

23 *a* On the same set of axes, sketch the graphs of $y = 2 \cos x$ and $y = \sin \frac{1}{2}x$ as x varies from 0 to 2π radians. [4, 4]

b From the graphs made in answer to *a*, determine the number of values of x, greater than 0 and less than 2π, for which $2 \cos x = \sin \frac{1}{2}x$. [2]

24 *a* In triangle ABC in which angle A is an obtuse angle, prove that $c = a \cos B + b \cos A$. [*Suggestion:* Draw the altitude to side c.] [8]

b Does this formula hold if angle A is (1) acute, (2) right? [1, 1]

Part III

Answer at least two questions from part III.

25 Find, to the *nearest integer*, the area of a regular polygon of 8 sides if one side is 36. [10]

26 A tree grows on a slope that is inclined 13° to the horizontal. At a point 70 feet down the slope from the foot of the tree, the tree subtends an angle of 35° 10′. Find to the *nearest foot* the height of the tree. [6, 4]

27 A bridge over a river is 250 feet long. A boat is anchored directly under the bridge. From the boat the angles of elevation of the ends of the bridge are 40° 30′ and 34° 20′. Find to the *nearest foot* the height of the bridge above the river. [5, 5]

28 *a* A boat sails from port P in the direction N 15° E for 27 miles to S. It then changes its course to N 85° E and sails on that course for 19 miles to R. Find to the *nearest degree* angle RPS. [4, 5]

b Find the direction of R from P. [1]

Fill in the following lines:

Name of pupil.....Name of school.....

Part I

Answer all questions in part I. Each correct answer will receive 2½ credits. No partial credit will be allowed.

1 Express in degrees an angle of $\frac{3}{5}\pi$ radians. 1.....

2 An angle of $1\frac{1}{2}$ radians at the center of a circle subtends an arc of 9 inches. Find the length of the radius in inches. 2.....

3 Express tan 124° as a function of a positive acute angle. 3.....

4 If $7 \sin A = 3$, express A as an inverse function. 4.....

5 The area of triangle ABC is 38. If $a = 25$ and $b = 8$, find the sine of angle C. 5.....

6 Find log .2356 6.....

7 Find cos 48° 43′ 7.....

8 Find to the *nearest minute* the positive acute angle for which $\log \tan A = 9.9016 - 10$ 8.....

9 In triangle ABC, $a = 5$, $b = 6$, $c = 10$. Find cos B. 9.....

10 In triangle ABC, $a = 8$, $\sin A = .4$ and $\sin B = .7$. Find b. 10.....

11 In triangle ABC, $b = 12$, $c = 6$, $A = 100°$. Find to the *nearest hundredth* $\tan \frac{1}{2}(B - C)$. 11.....

12 Find the positive acute value of x for which $\sec^2 x - 4 = 0$. 12.....

13 If $\tan x = \frac{2}{3}$ and $\tan y = \frac{1}{2}$, find the value of $\tan (x + y)$. 13.....

14 If $\cos x = a$, express $\cos 2x$ in terms of a. 14.....

[OVER]

15 If $\cos x = \frac{8}{5}\frac{1}{0}$, find the positive value of $\cos \frac{1}{2} x$. 15.....

16 If A is a positive acute angle, express $\sin A$ in terms of $\tan A$. 16.....

Directions (17–20) — Indicate the correct completion for *each* of the following by writing on the line at the right the letter *a, b* or *c*.

17 The statement $1 - \cos 2x = 2 \sin^2 x$ is true for (*a*) no value of x (*b*) certain values of x (*c*) all values of x 17.....

18 Using the data $b = 12$, $a = 10$, $A = 150°$, there can be constructed (*a*) no triangle (*b*) one triangle (*c*) two triangles 18.....

19 $\sin 70° - \sin 10°$ is equal to (*a*) $\sqrt{3} \sin 40°$ (*b*) $\cos 40°$ (*c*) $-\cos 40°$ 19.....

20 As angle x increases, both $\sec x$ and $\csc x$ decrease in the (*a*) second quadrant (*b*) third quadrant (*c*) fourth quadrant 20.....