The University of the State of New York

TRIGONOMETRY

Thursday, January 24, 1952 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (a) name of school where you have studied, (b) number of weeks and recitations a week in trigonometry.

The minimum time requirement is four or five recitations a week for half a school year, or the equivalent.

Answer five questions from parts II and III, including at least two questions from each part.

Part II

Answer at least two questions from part II.

21 Find to the *nearest degree* all values of θ greater than $0°$ and less than $180°$ that satisfy the equation $2 \sec \theta - 4 \cos \theta + 7 \tan \theta = 0$. [10]

22 Derive the formula for $\cos (x + y)$ in which x, y and $(x + y)$ are positive acute angles. [10]
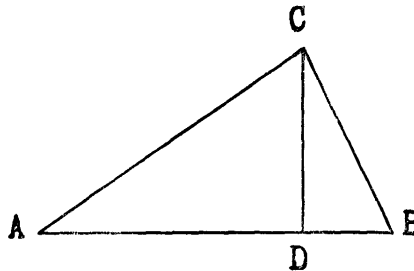
23 a On one set of axes sketch the graphs of $y = 2 \sin x$ and $y = \cos 2x$ from 0 to π radians. [4, 4]

　 b From the graphs made in answer to a, determine the number of values of x greater than 0 and less than π radians for which $2 \sin x = \cos 2x$. [2]

24 a Prove the identity: $2 \cos x - \dfrac{\cos 2x}{\cos x} = \sec x$　[4]

　 b In the figure at the right CD is perpendicular to AB. Derive the formula

$$CD = \frac{AB}{\cot A + \cot B}\qquad [6]$$

[1]　　　　　　　　　　　　　　　　　　　　　　　　[OVER]

Part III

Answer at least two questions from part III.

25 In triangle ABC, $a = 374$, $b = 224$, and $c = 316$. Find A to the *nearest minute*. [10]

26 A breakwater running north and south is 1460 ft. long. From the northern end of the breakwater the bearing of a boat is N 63° 13' W and from the southern end the bearing is N 24° 13' W. Find to the *nearest ten feet* the distance from the boat to the nearer end of the breakwater. [5, 5]

27 A building 27 feet high stands on level ground at the foot of a hill. From the top of the hill the angles of depression of the top and of the bottom of the building are 42° 10' and 49° 20' respectively. Find to the *nearest foot* the height of the hill. [4, 6]

28 Two forces of 279 lb. and 326 lb. act on a body at an angle of 65° with each other. Find to the *nearest degree* the angle formed by the lines of action of the resultant and the larger force. [10]

Fill in the following lines:

Name of pupil...Name of school...

Part I

Answer all questions in part I. Each correct answer will receive 2½ credits. No partial credit will be allowed.

1 Express in degrees an angle of $\dfrac{7\pi}{6}$ radians.

1................

2 In a circle whose radius is 12 inches a central angle subtends an arc of 16 inches. Find the number of radians in the angle.

2................

3 Express sin 290° as a function of a positive acute angle.

3................

4 Find the logarithm of .2674

4................

5 Find the number whose logarithm is 0.7646

5................

6 Find cos 40° 23′

6................

7 Find to the *nearest minute* the positive acute angle A for which log sin $A = 9.8729 — 10$.

7................

8 Two sides of a parallelogram are 12 and 15 and the included angle is 54°. Find to the *nearest integer* the area of the parallelogram.

8................

9 In triangle ABC, $a = 6$, $b = 7$ and cos $C = \frac{1}{4}$. Find c.

9................

10 If $A = \tan^{-1}\dfrac{4}{\sqrt{33}}$, find the positive value of sin A.

10................

11 If A is a positive acute angle, express tan A in terms of cos A.

11................

12 In triangle DEF, $d = 12$, sin $D = \frac{5}{7}$ and sin $F = \frac{5}{6}$. Find f.

12................

13 In triangle ABC, $a = 9$, $b = 7$ and $C = 80°$. Find tan $\frac{1}{2}(A — B)$ to the *nearest hundredth*.

13................

[3]

14 If x and y are acute angles and $\sin x = \dfrac{6}{\sqrt{85}}$ and $\sin y = \dfrac{2}{\sqrt{85}}$, find $\sin (x - y)$.

14................

15 If $\cos \theta = \frac{17}{49}$, find the positive value of $\sin \frac{1}{2} \theta$.

15................

16 Express $\cos 5x - \cos 3x$ as a product of two functions.

16................

17 Find the smallest positive value of x for which $3 \cot x = \tan x$.

17................

Directions (questions 18–20) — Indicate the correct completion for *each* of the following by writing on the line at the right the letter a, b or c.

18 Using the data $A = 27°$, $b = 12$ and $a = 5.7$, it is possible to construct
(*a*) no triangle   (*b*) only one triangle   (*c*) two triangles

18......

19 For all values of x, $\tan x$ is equal to
(*a*) $\dfrac{1}{\cot (-x)}$ (*b*) $\dfrac{\sin x}{\cos (-x)}$ (*c*) $\dfrac{\sin (-x)}{\cos x}$

19......

20 The statement $2 \sin \frac{3}{2} x \cos \frac{3}{2} x = \sin 3x$ is true for   (*a*) no values of x
(*b*) only certain values of x (*c*) all values of x

20......