The University of the State of New York

311TH HIGH SCHOOL EXAMINATION

TRIGONOMETRY

Thursday, January 25, 1951 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in trigonometry.

The minimum time requirement is four or five recitations a week for half a school year, or the equivalent.

Answer five questions from parts II and III, including at least two questions from each part.

Part II

Answer at least two questions from part II.

21 *a* Starting with the formulas for $\sin(x + y)$ and $\cos(x + y)$, derive the formula for $\tan(x + y)$ in terms of $\tan x$ and $\tan y$. [5]

b Prove the identity: $\cot A = \dfrac{\cos 2A + \cos A + 1}{\sin 2A + \sin A}$ [5]

22 *a* On the same set of axes sketch the graphs of $y = 2\cos x$ and $y = \cos 2x$ as x varies from 0 to π radians inclusive. [4, 4]

b From the graphs made in answer to *a*, determine the number of values of x between 0 and π radians that satisfy the equation $2\cos x = \cos 2x$. [2]

23 Find, to the *nearest degree*, all values of x between 0° and 360° which satisfy the equation $3\cos^2 x - 5\sin x - 1 = 0$. [10]

24 From two ships due east of a lighthouse and in line with its foot, the angles of elevation of the top of the lighthouse are x and y, with x greater than y. The distance between the ships is m. Show that the distance d from the lighthouse to the nearer ship is given by the formula:

$$d = \frac{m\cos x \sin y}{\sin(x - y)}$$ [10]

Part III

Answer at least two questions from part III.

25 In triangle ABC, $a = 72$, $b = 53$ and $c = 67$. Find A to the *nearest minute*. [10]

26 A building lot is in the form of a quadrilateral $ABCD$. Diagonal AC is perpendicular to side AB. Find, to the *nearest square foot*, the area of the lot if $AB = 62$ ft., $AC = 75$ ft., $AD = 58$ ft. and angle $BAD = 132°$. [4, 6]

27 To find the width of a river, a surveyor selected two points A and B along one shore and another point P on the opposite shore. He made the following measurements: angle $PAB = 53°$, angle $PBA = 80°$ and $AB = 500$ ft. Find, to the *nearest foot*, the width of the river at point P. [4, 6]

28 A ship sails from A to B, a distance of 25 miles, in the direction S 55° E and thence from B to C, a distance of 42 miles, in the direction N 23° E. Find, to the *nearest degree*, the direction of C from A. [5, 5]

Fill in the following lines:

Name of pupil..... ... .Name of school .....

Part I

Answer all questions in part I. Each correct answer will receive 2½ credits. No partial credit will be allowed.

1 Express 240° in radians. [Answer may be left in terms of π.] 1.....

2 Find the value of $\sin \frac{\pi}{6}$ 2.....

3 Find the value of $\tan (-135°)$. 3.....

4 Express sec 320° as a function of a positive acute angle. 4.....

5 Find log cos 53° 42′ 5.....

6 Find, to the *nearest minute,* the positive acute angle whose sine is .5360 6.....

7 Find the logarithm of 703.2 7.....

8 If $5 \cos A = 3$, express A as an inverse function. 8.....

9 Find the smallest positive value of x that satisfies the equation $\cos 3x = \sin 54°$. 9.....

10 In which quadrant does an angle lie if its sine is positive and its tangent is negative? 10.....

11 Express $\cot^2 A$ in terms of $\sin A$. 11.....

12 If $\cos x = a$ and x is acute, express $\cos \frac{1}{2}x$ in terms of a. 12.....

13 An artillery spotter in a plane at an altitude of 1000 ft. observes the angle of depression of an enemy tank to be 21°. How far, to the *nearest foot,* is the enemy tank from the point on the ground directly below the spotter? 13.....

14 In triangle RST, $r = 8$, $\sin R = \frac{2}{3}$ and $\sin S = \frac{3}{4}$; find s. 14.....

15 In triangle ABC, $a = 4$, $b = 7$, $c = 9$. Find $\cos B$. 15.....

16 In triangle ABC, $a = 9$, $b = 6$, $C = 120°$. Find $\tan \frac{1}{2}(A - B)$. [Answer may be left in radical form.] 16.....

17 Express $\sin 50° - \sin 20°$ as the product of two functions. 17.....

Directions (questions 18–20) — Indicate the correct answer to each question by writing the letter *a*, *b* or *c* on the line at the right.

18 Using the data $a = 5$, $b = 10$ and $A = 40°$, it is possible to construct (*a*) only one triangle (*b*) two triangles (*c*) no triangle 18.....

19 As angle A increases from 90° to 180°, $\cos A$ (*a*) decreases from 1 to 0 (*b*) increases from −1 to 0 (*c*) decreases from 0 to −1 19.....

20 The minimum value of $2 \sin x$ is (*a*) 0 (*b*) −1 (*c*) −2 20.....