The University of the State of New York

308TH HIGH SCHOOL EXAMINATION

TRIGONOMETRY

Thursday, January 26, 1950—9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in trigonometry.

The minimum time requirement is four or five recitations a week for half a school year, or the equivalent.

Answer five questions from parts II and III, including at least two questions from each part.

Part II

Answer at least two questions from part II.

21 *a* Prove the identity $\frac{\sin 2A}{1 + \cos 2A} = \tan A$ [5]

b Find, to the *nearest degree,* the positive acute angle which satisfies the equation $\cos^2 x - \sin^2 x = \frac{7}{25}$ [5]

22 Derive the formula for $\sin (x + y)$ where x and y are positive and $(x + y)$ is acute. [10]

23 *a* On the same set of axes sketch the graphs of $y = \cos x$ and $y = \sin \frac{x}{2}$ as x varies from 0 to 2π radians. [3, 6]

b From the graphs made in answer to *a,* determine the number of values of x between 0 and 2π radians which satisfy the equation $\sin \frac{x}{2} = \cos x$ [1]

24 In $\triangle ABC$, the bisector of angle B meets AC in D. If AD is represented by m, show that $DC = \frac{m \sin A}{\sin C}$. [10]

Part III

Answer at least two questions from part III.

25 The sides of a triangle are 60, 28 and 40.

a Find the area of the triangle. [4]

b Using the result obtained in answer to *a*, find, to the *nearest integer,* the altitude on side 28. [6]

26 A body is acted upon by two forces of 225 lb. and 210 lb. The angle between the lines of action of the forces is 75° 40′. Find, to the *nearest minute,* the angle formed by the lines of action of the resultant and the 210-lb. force. [3, 7]

27 Find, to the *nearest inch,* the side of a regular pentagon whose area is 275 sq. in. [10]

28 A vertical tower 120 feet high stands on top of a hill that has a slope of 20° to the horizontal. From the top of the tower the angle of depression of a point on the side of the hill is 47°. Find, to the *nearest foot,* the distance, measured along the side of the hill, of this point from the foot of the tower. [4, 6]

Fill in the following lines:

Name of pupil..... .....Name of school.....

Part I

Answer all questions in part I. Each correct answer will receive 2½ credits. No partial credit will be allowed.

1 Find the logarithm of 2.768 1.....

2 Find the number whose logarithm is 1.8099 2.....

3 Find log cos 24° 22′ 3.....

4 Find tan 66° 36′ 4.....

5 Express in radians an angle of 140°. [Answer may be left in terms of π.] 5.....

6 Express sin 289° as a function of a positive angle less than 45°. 6.....

7 Find the positive value of $\cot \sin^{-1} \frac{\sqrt{2}}{2}$ 7.....

8 In $\triangle ABC$, $a = 9$, $b = 5$, $C = 60°$. Find c to the *nearest integer.* 8.....

9 In $\triangle ABC$, $a = 12$, $b = 15$, $C = 150°$. Find the area of $\triangle ABC$. 9.....

10 A is 200 miles N 60° W of B. C is due south of A and also due west of B. How far is A from C? 10.....

11 If $\sin x = \frac{2}{\sqrt{5}}$ and $\cos y = \frac{3}{\sqrt{13}}$, and x and y are positive acute angles, find $\sin (x - y)$. [Answer may be left in radical form.] 11.....

12 If $\cos x = \frac{3}{5}$ and x is an acute angle, find $\tan^2 \frac{x}{2}$ 12.....

13 In $\triangle ABC$, $a = 10$, $b = 8$, $C = 60°$; find $\tan \frac{A - B}{2}$. [Answer may be left in radical form.] 13.....

14 Express $\frac{2 \tan x}{\sec^2 x}$ as a single function of $2x$. 14.....

15 In $\triangle ABC$, angle A is acute. If $\sin A = \cos B$, find $\sin C$. 15.....

16 A root of the equation $\tan x + \cot 2x = \csc x$ is 30°. Is this statement *true* or is it *false?* 16.....

17 $\sin (-A) = -\sin A$. Is this statement *true* or is it *false?* 17.....

Directions (questions 18–20) — Indicate the correct answer to *each* question by writing on the line at the right the letter *a, b* or *c.*

18 Sin 55° — sin 15° equals (*a*) sin 40° (*b*) 2 cos 35° sin 20° (*c*) 2 sin 35° cos 20° 18.....

19 If both sin x and cos x increase as x increases, then x must be an angle in quadrant (*a*)two (*b*)three (*c*)four 19.....

20 Using the data $A = 125°$, $a = 50$, $b = 35$, it is possible to construct (*a*)only one triangle (*b*)two different triangles (*c*)no triangle 20.....