TRIGONOMETRY

Thursday, January 25, 1945 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II, III and IV (a) name of school where you have studied, (b) number of weeks and recitations a week in trigonometry.

The minimum time requirement is five recitations a week for half a school year, or the equivalent.

Answer five questions from parts II, III and IV, including at least one question from each part.

Part II

Answer at least one question from part II.

21 a Derive the law of sines for the acute plane triangle. [6]

b Starting with the formula for $\cos (x + y)$, derive the formula for $\cos 2x$ in terms of $\sin x$. [4]

22 a Prove that the expression $(\tan B + \cot B) \sin B \cos B$ equals 1. [3]

b Solve the equation $\sin^2 y - 2 \cos y + \frac{1}{4} = 0$ for all values of y between 0° and 360°. Check *one* value. [5, 2]

23 a On the same set of axes, draw the graphs of $y = \cos x$ and $y = \sin 2x$ as x varies from 0 to π radians inclusive at intervals of $\frac{\pi}{6}$ radians. [3, 5]

b Indicate on the graphs, by means of capital letters, the points whose abscissas give solutions of the equation $\cos x = \sin 2x$. [2]

24 A valley is crossed by a bridge AB whose length is d; C is a point in the valley directly below the bridge. The angles of depression of C at A and B are s and t, as shown in the drawing. In terms of s, t and d, derive a formula for the height h of the bridge above C. [10]

[1] [OVER]
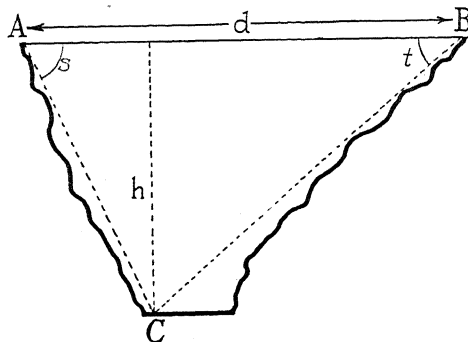
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Part III

Answer at least one question from part III.

25 In triangle *ABC*, $a = 328$, $b = 321$ and $c = 295$. Find angle *B* correct to the *nearest minute*. [10]

26 From a point *C* at sea level, the angle of elevation of a mountain peak *B* is 30°. An aviator at *A*, 4325 feet directly above *C*, finds that angle *BAC* is 43°. Find, correct to the *nearest foot*, the height of the mountain peak above sea level. [10]

27 A ship sails 23 miles on a course N 15° E and then 15 miles on a course N 78° E. In what direction, correct to the *nearest minute*, is the ship from the starting point? [10]

Part IV

Answer at least one question from part IV.

28 In spherical triangle *ABC*, $A = 20° \ 30'$, $B = 84° \ 40'$, $c = 90°$. Find *C*. [10]

29 Find the great circle distance in statute miles between London (Lat. 51° 31′ N, Long. 0° 6′ W) and Berlin (Lat. 52° 32′ N, Long. 13° 24′ E). [1 nautical mile = 1.152 statute miles] [10]

Fill in the following lines:

Name of school...Name of pupil...

Part I

Answer all questions in part I. Each correct answer will receive 2½ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1 Express cos 87° as a function of a positive angle less than 45°. 1................

2 Express cot $(180° + A)$ as a function of A. 2................

3 Find the value of sin 163° 3................

4 Find log sin 61° 23′ 4................

5 Find acute angle A correct to the *nearest minute,* if log cos $A =$ 9.9020 — 10 5................

6 If A is a positive acute angle and sec $A = \frac{17}{8}$, find tan A. 6................

7 Express in degrees an angle of $\frac{\pi}{3}$ radians. 7................

8 A circular arc of 30 feet subtends an angle of four radians at the center of its circle. Find the radius of the circle. 8................

9 Express in degrees an angle of 40 mils. 9................

10 In which quadrants may the terminal side of an angle lie if its tangent is negative? 10................

11 Express tan $2x$ in terms of tan x. 11................

12 If A is an angle in the first quadrant, express tan A in terms of cos A. 12................

13 Complete the formula sin $(A + B) = \dots$ 13................

14 In right spherical triangle ABC, in which C is the right angle, c and b are known. Write the formula that should be used to find B. 14................

15 Complete the following statement: In the solution of a right spherical triangle ABC, in which C is the right angle, an ambiguous case arises when the given parts are a and .... 15................

16 Two sides of a parallelogram are 6 and 10 and the included angle is 25°. Find, correct to the *nearest integer,* the area of the parallelogram. 16................

17 In triangle ABC, $a = 4$, $b = 5$ and $c = 6$. Find the value of cos A. 17................

18 Find the positive acute angle which satisfies the equation $\tan^2 x - 3 = 0$ 18................

19 In plane triangle ABC, angle A is acute. If a is less than b and a is greater than b sin A, how many solutions has the triangle? 19................

20 In spherical triangle ABC, if $a = 125°$, $c = 80°$ and $C = 90°$, in what quadrant is b? 20................