The University of the State of New York

290TH HIGH SCHOOL EXAMINATION

TRIGONOMETRY

Thursday, January 20, 1944 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II, III and IV (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in trigonometry.

The minimum time requirement is five recitations a week for half a school year, or the equivalent.

Answer five questions from parts II, III and IV, including at least one question from each part.

Part II

Answer at least one question from part II.

21 *a* Derive the law of cosines for plane triangles. Consider only the case in which the triangle is acute. [7]

b The sides of a plane triangle are 4, 5 and 7. Find the largest angle of the triangle correct to the *nearest degree*. [3]

22 Find, correct to the *nearest minute*, the angles between 0° and 360° which satisfy the equation $5 \cos^2 x - 8 \sin x - 1 = 0$ [10]

23 *a* Draw the graph of $y = \sin 2x$ as x varies from 0 to π inclusive at intervals of $\frac{\pi}{6}$. [6]

b Using the same set of axes as in *a*, draw the graphs of $y = \frac{1}{2}$ and $y = -\frac{1}{2}$ [2]

c By means of the graphs made in answer to *a* and *b*, determine the number of values of x between 0 and π for which $\sin 2x = \pm\frac{1}{2}$ [2]

24 Prove that the area K of a regular polygon of n sides inscribed in a circle of radius R is given by the formula $K = n R^2 \sin \frac{180°}{n} \times \cos \frac{180°}{n}$ [10]

Part III

Answer at least one question from part III.

25 Two observers, A and B, at the ends of a level base line 1000 yards long, measure angles from the base line to a gun emplacement G. If angle $BAG = 37° 20'$ and angle $ABG = 62° 30'$, find BG. [10]

26 Two airplane spotters are at the ends, A and B, of a level base line 5280 feet long. A plane crosses the base line and, at the instant of crossing, the spotter at A measures its angle of elevation as 48° 30′ and the spotter at B measures its angle of elevation as 51° 10′. Find, correct to the *nearest foot*, the height of the plane above the ground. [10]

27 Two forces of 175 lb. and 35 lb. act on a body. The angle between the lines of action of the forces is 30°. Find the magnitude and the direction of the resultant. [10]

Part IV

Answer at least one question from part IV.

28 Given a right spherical triangle, ABC, with the right angle at C; if side $a = 58° 10'$ and angle $B = 73° 10'$, find angle A correct to the *nearest minute*. [10]

29 Dutch Harbor is situated at latitude 53° 53′ N., longitude 166° 35′ W., and Tokyo at latitude 35° 39′ N., longitude 139° 45′ E. Find, correct to the *nearest nautical mile*, the distance from Dutch Harbor to Tokyo. [1 nautical mile = 1 minute of arc of a great circle on the surface of the earth] [10]

Fill in the following lines:

Name of school.....Name of pupil.....

Part I

Answer all questions in this part. Each correct answer will receive 2½ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1 Find the logarithm of 0.03742 1.....

2 Find, correct to the *nearest tenth*, the number whose logarithm is 2.8200 2.....

3 Find log tan 27° 13′ 3.....

4 Log cos $x = 9.7425 - 10$. Find x correct to the *nearest minute*. 4.....

5 Express tan 215° as a function of a positive angle less than 45°. 5.....

6 Express $\frac{5}{6}\pi$ radians in degrees. 6.....

7 Express in degrees an angle of 320 mils. 7.....

8 Express cos $2x$ in terms of cos x. 8.....

9 Express $\sin^2 \frac{x}{2}$ in terms of cos x. 9.....

10 Express (cos 110° — cos 50°) as a function of 80°. 10.....

11 Express $\tan^2 B$ in terms of sec B. 11.....

12 Find the positive acute angle which satisfies the equation $2\tan^2 x + \tan x - 3 = 0$ 12.....

13 The side of a regular pentagon is 8. Find the apothem of the pentagon correct to the *nearest tenth*. 13.....

14 If $\cos x = \frac{4}{5}$ and x is in the fourth quadrant, find cot x. 14.....

15 If $\sin x = -\frac{1}{\sqrt{5}}$ and x is in the third quadrant, find sin $2x$. 15.....

16 In right spherical triangle ABC in which C is the right angle, angles A and B are known. Write the formula that should be used to find c. 16.....

Directions (questions 17–20) — Indicate the correct answer to *each* question by writing the letter a, b or c on the line at the right.

17 The oblique plane triangle in which $B = 35°$, $b = 5$ and $c = 8$ has (*a*) two solutions, (*b*) one solution, (*c*) no solution 17.....

18 Cos $(x + y)$ is equal to (*a*) $\sin x \cos y + \cos x \sin y$, (*b*) $\cos x \cos y - \sin x \sin y$, (*c*) $\cos x \cos y + \sin x \sin y$ 18.....

19 The area of the oblique plane triangle ABC having a, b and C given, is (*a*) $\frac{1}{2}ab$, (*b*) $\frac{1}{2}ab \cos C$, (*c*) $\frac{1}{2}ab \sin C$ 19.....

20 If in right spherical triangle ABC, A is greater than 90°, then a is (*a*) less than 90°, (*b*) equal to 90°, (*c*) greater than 90° 20.....