The University of the State of New York

269TH HIGH SCHOOL EXAMINATION

MATHEMATICS — Third Year

Wednesday, June 16, 1937 — 9.15 a. m. to 12.15 p. m., only

Instructions

Do not open this sheet until the signal is given.

Group I

This group is to be done first and the maximum time allowed for it is one and one half hours.

If you finish group I before the signal to stop is given you may begin group II. However, it is advisable to look your work over carefully before proceeding, since *no credit will be given any answer in group I which is not correct and in its simplest form.*

When the signal to stop is given at the close of the one and one half hour period, work on group I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

Groups II and III

Write at top of first page of answer paper to groups II and III (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in mathematics third year.

The minimum time requirement is five recitations a week for a school year after the completion of elementary algebra.

In this examination the customary lettering is used. A, B and C represent the angles of a triangle ABC; a, b and c represent the respective opposite sides. In a right triangle, C represents the right angle.

Give special attention to neatness and arrangement of work.

The use of the slide rule will be allowed for checking but all computations with tables must be shown on the answer paper.

Answer *five* questions from these two groups, including at least *two* questions from each group.

N 2 3

Fill in the following lines:

Name of school.....Name of pupil.....

Detach this sheet and hand it in at the close of the one and one half hour period.

Group I

Answer all questions in this group. Each correct answer will receive 2½ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1 Find log 24.38 *Ans*.....

2 Find log tan 28° 33′ *Ans*.....

3 Two streets intersect at an angle of 62°. A triangular lot has a frontage of 40 feet on one street and 50 feet on the other. Find, correct to the *nearest square foot*, the number of square feet of land in the lot. *Ans*.....

4 The base of an isosceles triangle is 20 and the vertex angle is 40°. Find, correct to the *nearest tenth*, the altitude on the base. *Ans*.....

5 Write the three factors of $4 \sin^3 x - \sin x$ *Ans*.....

6 Given the formula $d = \sqrt{\frac{Tw}{l\,p}}$; express T as a function of d, l, p and w. *Ans*.....

7 Combine into a single term $\sqrt{24\frac{1}{2}} - \sqrt{18}$ *Ans*.....

8 Write the equation of the straight line whose slope is 3 and which passes through the point $(0, -1)$. *Ans*.....

9 Write in the form $x^2 + px + q = 0$, the equation whose roots are 3 and −2. *Ans*.....

10 Simplify $\dfrac{\frac{1}{x} + \frac{1}{y}}{\frac{1}{x} - \frac{1}{y}}$ *Ans*.....

11 Write the *third* term of the expansion $(x + \frac{1}{x})^5$. *Ans*.....

12 Find the length of the arc intercepted by a central angle of one radian in a circle whose radius is 20. *Ans*.....

13 Find the sum of the first 15 terms of the progression $3, 2\frac{1}{2}, 2, \ldots$ *Ans*.....

14 Write the positive value of $\sin(\tan^{-1} \frac{\sqrt{3}}{3})$ *Ans*.....

15 In triangle ABC, $a = 4$, $b = 5$, $c = 8$; find the cosine of the smallest angle of the triangle. *Ans*.....

16 In triangle ABC, $A = 30°$, $B = 45°$, $a = 2\sqrt{2}$; find b. *Ans*.....

17 Express $\cot^2 A + 1$ as a function of $\sin A$. *Ans*.....

18 If $4^x = \frac{1}{16}$, what is the value of x? *Ans*.....

19 Insert *two* geometric means between 7 and 189. *Ans*.....

20 Find the positive values of x and y that satisfy the equations $x^2 + y^2 = 160$ and $y = 3x$ *Ans*.....

See instructions for groups II and III on page 1.

Answer five questions from groups II and III, including at least two questions from each group.

Group II

Answer at least two questions from this group.

21 Find, correct to the *nearest tenth*, the roots of the equation $x^2 + 7x - 9 = 0$ [10]

22 The formula $D = \sqrt[3]{\frac{38\,H}{R}}$ gives the diameter in inches of a steel shaft which will transmit H horsepower when revolving at R revolutions per minute. Find, correct to the *nearest hundredth of an inch*, the value of D that will transmit 236 horsepower at 1150 revolutions per minute. [10]

23 A man travels by motorboat 24 miles up a river and returns immediately to the starting point. The time required for the round trip is 5 hours. If the rate of the stream is 2 miles per hour, find the rate of the motorboat in still water. [7, 3]

24 A man invested a part of his money so that it yielded 4%. He invested the remainder in an enterprise which lost 2%. His net income from these two sources for the first year was \$60. If in making his investments the amounts of money had been interchanged, his net income would have been \$12. How much money was invested in each enterprise? [7, 3]

25 *a* Draw the graph of $y = x^2 - 2x + 3$ from $x = -1$ to $x = 3$ inclusive. [5]

b Write the equation of the axis of symmetry of the graph made in answer to *a*. [3]

c Is the axis of symmetry for the curve represented by $y = x^2 - 2x$ the same as the axis of symmetry for the curve made in answer to *a*? [2]

*26 Solve the following set of equations for x, y and z:

$$2x - 3y - z = 6$$
$$x + 5y + 4z = -5$$
$$x + 2y + z = 1$$ [10]

Group III

Answer at least two questions from this group.

27 A surveyor lays out a triangular plot ABC. Angle A is 33° 20′, angle C is 117° 40′ and side AC is 380 feet. Find, correct to the *nearest foot*, the length of AB. [5, 5]

28 In triangle RST, angle $R = 54°\ 10'$, side $RS = 126$ feet and side $RT = 94$ feet; find angle T correct to the *nearest minute*. [3, 7]

29 *a* Solve the following equation for all positive values of x less than 180°:

$$\sin x + \cos 2x = 4 \sin^2 x$$ [6]

b Prove the identity: $\sec x - \tan x \sin x = \cos x$ [4]

* This question is based on one of the optional topics in the syllabus.