

The University of the State of New York

263D HIGH SCHOOL EXAMINATION

**MATHEMATICS — Third Year**

Thursday, June 20, 1935 — 9.15 a. m. to 12.15 p. m., only

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**Instructions**

*Do not open this sheet until the signal is given.*

**Group I**

*This group is to be done first and the maximum time allowed for it is one and one half hours.*

If you finish group I before the signal to stop is given you may begin group II. However, it is advisable to look your work over carefully before proceeding, since *no credit will be given any answer in group I which is not correct and in its simplest form.*

When the signal to stop is given at the close of the one and one half hour period, work on group I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

**Groups II and III**

Write at top of first page of answer paper to groups II and III (a) name of school where you have studied, (b) number of weeks and recitations a week in mathematics third year.

The minimum time requirement is five recitations a week for a school year after the completion of elementary algebra.

In this examination the customary lettering is used.  $A$ ,  $B$  and  $C$  represent the angles of a triangle  $ABC$ ;  $a$ ,  $b$  and  $c$  represent the respective opposite sides. In a right triangle,  $C$  represents the right angle.

Give special attention to neatness and arrangement of work.

The use of the slide rule will be allowed for checking but all computations with tables must be shown on the answer paper.



Fill in the following lines:

Name of school.....Name of pupil.....

Detach this sheet and hand it in at the close of the one and one half hour period.

Group I

Answer all questions in this group. Each correct answer will receive  $2\frac{1}{2}$  credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

- 1 Factor  $3 \cos^2 x - 2 \cos x - 5$  Ans.....
- 2 Which is greater, 1 radian or  $60^\circ$ ? Ans.....
- 3 Find  $\log \cos 62^\circ 12'$  Ans.....
- 4 Find, correct to the nearest minute, the smallest positive value of  $A$  if  $\log \tan A = 9.8651 - 10$  Ans.....
- 5 Write in the form  $x^2 + px + q = 0$  the equation whose roots are 4 and  $-2$ . Ans.....
- 6 What is the value of  $\sin(\sin^{-1} \frac{1}{3})$ ? Ans.....
- 7 Express  $\sec 290^\circ$  as a trigonometric function of a positive angle less than  $45^\circ$ . Ans.....
- 8 Solve for  $f$  the formula  $\frac{1}{f} = \frac{1}{a} + \frac{1}{c}$  Ans.....
- 9 Write the equation of the straight line whose slope is 3 and which passes through the point  $(4, 2)$ . Ans.....
- 10 Write the first three terms of the expansion  $(x - 2y)^7$  Ans.....
- 11 Solve for  $x$  the equation  $x + \sqrt{x^2 + 1} = 3$  Ans.....
- 12 In the triangle  $ABC$ ,  $a = 5$ ,  $c = 8$ ,  $B = 60^\circ$ ; find  $b$ . Ans.....
- 13 Insert three positive geometric means between 3 and  $\frac{1}{3}$ . Ans.....
- 14 Write the equation of the straight line passing through the points whose coordinates are given in the following table: Ans.....

$x$	0	3	5	8	...
$y$	-3	6	12	21	...

- 15 If  $\tan x = \sqrt{3}$  and  $\sin x$  is negative, find  $\cos x$ . Ans.....
- 16 Express  $\sin x$  in terms of  $\tan x$  when  $x$  is an angle in the first quadrant. Ans.....
- 17 In the right triangle  $ABC$ ,  $AB = 12$ ,  $\sin A = \frac{1}{3}$ ; find  $AC$ . [Answer may be left in radical form.] Ans.....
- 18 In the expression  $y = (1 + \sin x)^{-2}$ , does  $y$  increase or decrease as  $x$  increases from  $0^\circ$  to  $90^\circ$ ? Ans.....
- 19 If taxicab rates are  $p$  cents for the first mile and  $m$  cents for each additional third of a mile, find the cost in cents of a ride of  $x$  miles, where  $x$  is any integer. Ans.....
- 20 If  $\tan x = \frac{3}{4}$  and  $x$  is an angle in the third quadrant, find  $\cos \frac{x}{2}$ . [Answer may be left in radical form.] Ans.....



See instructions for groups II and III on page 1.

Group II

Answer three questions from this group.

- 21 Solve the following system of equations, correctly group your answers and check *one* set:

$$\begin{aligned} 2x^2 - 3y^2 + xy &= 0 \\ x - 3y &= 9 \end{aligned} \quad [7, 2, 1]$$

- 22 If \$500 is invested at 4%, interest compounded semiannually, what will the amount be at the end of 8 years?  $[A = P(1 + \frac{r}{2})^{2n}]$  [10]

- 23 An automobile radiator has a capacity of 16 quarts. It is filled with a 12½% solution of alcohol and water. How many quarts of the solution must be drained off and replaced by alcohol to make a 50% solution? [10]

- 24 The sum of three integral numbers in arithmetic progression is 9. The largest number is 7 times the square of the smallest. Find the numbers. [10]

- 25 a Draw the graph of  $y = 2x^2 + 4x + 2$  from  $x = -3$  to  $x = 2$  inclusive. [6]

- b From the graph made in answer to a, determine the number of real roots of the equation  $2x^2 + 4x + 2 = 0$  [2]

- c Give the equation of the axis of symmetry for the graph of  $y = 2x^2 + 4x + 2$  [2]

- \*26 Solve the following set of equations for  $x$ ,  $y$  and  $z$ :

$$\begin{aligned} 2x - 3y - 5z &= 5 \\ 3x + 4y + 3z &= 12 \\ x - 2y - 2z &= 0 \end{aligned} \quad [10]$$

Group III

Answer two questions from this group.

- 27 a Prove the identity:  $\frac{\tan 2x}{2 \tan x} = \frac{\cot^2 x}{\cot^2 x - 1}$  [4]

- b Solve for values of  $x$  between  $0^\circ$  and  $180^\circ$  that satisfy the equation  $2 - 3 \sin x = \cos 2x$  [6]

- 28 Solve the triangle  $ABC$ , in which  $a = 36.73$ ,  $b = 42.60$ ,  $B = 24^\circ 30'$  [10]

- 29 At a point 3.59 miles from one end of a lake and 5.75 miles from the other end, the lake subtends an angle of  $78^\circ 40'$ ; find the length of the lake correct to the *nearest hundredth of a mile*. [10]

\* This question is based on one of the optional topics in the syllabus.

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