The University of the State of New York

257TH HIGH SCHOOL EXAMINATION

MATHEMATICS — Third Year

Thursday, June 22, 1933 — 9.15 a. m. to 12.15 p. m., only

Instructions

Do not open this sheet until the signal is given.

Answer all questions in part I and five questions from part II.

Part I is to be done first and the maximum time to be allowed for this part is one and one half hours. Merely write the answer to each question in the space at the right; no work need be shown.

If you finish part I before the signal to stop is given you may begin part II. However, it is advisable to look your work over carefully before proceeding to part II, since *no credit will be given any answer in part I which is not correct and reduced to its simplest form.*

When the signal to stop is given at the close of the one and one half hour period, work on part I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

In this examination the customary lettering is used. A, B and C represent the angles of a triangle ABC; a, b and c represent the respective opposite sides. In a right triangle, C represents the right angle.

Give special attention to neatness and arrangement of work.

In both parts of this examination the use of the slide rule will be allowed for checking; in part II all computations with tables must be shown on the answer paper.

216

MATHEMATICS — Third Year

Thursday, June 22, 1933

Fill in the following lines:

Name of school.....Name of pupil.....

Detach this sheet and hand it in at the close of the one and one half hour period.

Part I

Answer all questions in this part. Each question has 2½ credits assigned to it; no partial credit should be allowed. Each answer must be reduced to its simplest form.

1 Find the value of $(3x)^0 + x^{-1}$, when $x = 8$ Ans.....

2 Express with a rational denominator the fraction $\frac{5}{3-\sqrt{3}}$ Ans.....

3 Find the sum of the fractions $\frac{b}{b^2-a^2}$ and $\frac{1}{a-b}$ Ans.....

4 Write the first *three* terms of the expansion $(x-a)^8$ Ans.....

5 What is the positive geometric mean between 4 and 25? Ans.....

6 When two cells are connected in series, the electric current in circuit can be computed from the formula $i = \frac{2E}{R+2r}$, in which E represents the voltage, r the internal resistance and R the external resistance. Solve this formula for r. Ans.....

7 The legs of a right triangle are a and b and the hypotenuse is c. Express c as a function of a and b; that is, express c in terms of a and b. Ans.....

8 Given the equation $x^2 - 6x + 9 = 0$; if the constant term 9 is increased, while the coefficients of the other terms remain the same, the roots of the resulting equation become (*a*) real and equal, (*b*) real and unequal or (*c*) imaginary. Which is correct, *a* or *b* or *c*? Ans.....

9 Write the equation of the straight line whose slope is ½ and whose *y*-intercept is 1. Ans.....

10 Find the radius of the circle whose center is at the point (0, 0) and which passes through the point (1, 2). Ans.....

11 Express sin 280° as a function of a positive acute angle. Ans.....

12 If cos A is negative and sin $A = -\frac{1}{2}$, what is the value of cot A? Ans.....

13 Find to the *nearest degree* angle A of triangle ABC if $a = 2$, $b = 3$ and $c = 4$ Ans.....

14 Find side a of triangle ABC if $b = 5$, $B = 30°$ and $A = 43°$ Ans.....

15 Express 135° in radian measure. [Answer may be left in terms of π.] Ans.....

16 Express the area K of triangle ABC as a function of a, b and C. Ans.....

33/2

Write at top of first page of answer paper to part II (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in (1) elementary algebra, (2) mathematics, third year.

The minimum time requirement is five recitations a week for a school year after the completion of elementary algebra.

Part II

Answer five questions from this part, selecting three questions from group I and two from group II.

Group I

Answer three questions from this group.

21 A cross-country team ran 6 miles at a constant rate and then returned at a rate 5 miles less per hour. At what rates did they run if they were 50 minutes longer in returning than in going? [7, 3]

22 A nurse prepared 10 ounces of a disinfectant that was 15% carbolic acid; how much water must she add to reduce it to a 6% solution? [7, 3]

23 A principal of $150, deposited in a trust company, bears interest at 4%, compounded semiannually. Using the formula $A = P\ (1 + \frac{r}{2})^{2n}$, compute to the *nearest dollar* the amount A, which had accumulated at the end of 12 years. [10]

24 Given the equation $x^2 + kx + k = 0$

a Express the discriminant d of the given equation as a function of k; that is, express d in terms of k. [3]

b Using the vertical axis to represent d and the horizontal axis to represent k, plot the graph of the function obtained in answer to *a*, using values of k from -1 to $+5$ inclusive. [5]

c On the graph made in answer to *b*, indicate the points at which the values of k will produce a zero discriminant. [2]

*25 Solve the following set of equations for x, y and z:

$$\frac{1}{x} + \frac{1}{y} = \frac{5}{6}$$

$$\frac{1}{y} + \frac{1}{z} = \frac{8}{15}$$

$$\frac{1}{x} + \frac{1}{z} = \frac{7}{10}$$ [10]

Group II

Answer two questions from this group.

26 To find the distance BC across a swamp, surveyors located a point A which was 80.7 feet from B and 110.3 feet from C. The angle BAC was measured and found to be 52° 0′; find the distance across the swamp. [10]

27 In the triangle ABC, $A = 54°\ 15'$, $B = 68°\ 20'$ and $AB = 10$; find the altitude on side AB. [10]

28 *a* If $x = \sin^{-1} \frac{3}{5}$ and $y = \cos^{-1} \frac{5}{13}$, both angles being acute, find by the use of the proper formula the value of $\sin(x + y)$. [7]

b Prove the following identity: $(\sin x + \cos x)^2 = 1 + \sin 2x$ [3]

* This question is based on one of the optional topics in the syllabus.

217