

MATHEMATICS — Third Year

Monday, January 23, 1939 — 9.15 a. m. to 12.15 p. m., only

Instructions

Do not open this sheet until the signal is given.

Group I

This group is to be done first and the maximum time allowed for it is one and one half hours.

Merely write the answer to each question in the space at the right; no work need be shown.

If you finish group I before the signal to stop is given you may begin group II. However, it is advisable to look your work over carefully before proceeding, since *no credit will be given any answer in group I which is not correct and in its simplest form.*

When the signal to stop is given at the close of the one and one half hour period, work on group I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

Groups II and III

Write at top of first page of answer paper to groups II and III (a) name of school where you have studied, (b) number of weeks and recitations a week in mathematics third year.

The minimum time requirement is five recitations a week for a school year after the completion of elementary algebra.

In this examination the customary lettering is used. A , B and C represent the angles of a triangle ABC ; a , b and c represent the respective opposite sides. In a right triangle, C represents the right angle.

Give special attention to neatness and arrangement of work.

The use of the slide rule will be allowed for checking but all computations with tables must be shown on the answer paper.

Answer five questions from these two groups, including at least two questions from each group.

Fill in the following lines:

Name of school.....Name of pupil.....

Detach this sheet and hand it in at the close of the one and one half hour period.

Group I

Answer all questions in this group. Each correct answer will receive $2\frac{1}{2}$ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

- 1 Find $\log \sin 20^\circ 43'$ 1.....
- 2 Factor $\cos^3 x - \cos x$ 2.....
- 3 Express $\sin 140^\circ$ as a function of a positive acute angle. 3.....
- 4 Write in the form $x^2 + px + q = 0$ the equation whose roots are $3 + \sqrt{2}$ and $3 - \sqrt{2}$ 4.....
- 5 Express $\frac{1}{\sqrt{5} - 1}$ as a fraction with rational denominator. 5.....
- 6 Write the expression for $\tan 2A$ in terms of $\tan A$. 6.....
- 7 Write the positive value of $\cos (\tan^{-1} 1)$ 7.....
- 8 Find the value of $27^{\frac{2}{3}} + 2^{-1}$ 8.....
- 9 How many different triangles are there in which $A = 20^\circ$, $a = 3$ and $b = 5$? 9.....
- 10 In the triangle ABC , find the value of c if $a = 3$, $b = 4$ and $\cos C = \frac{1}{3}$ 10.....
- 11 Find the arithmetic mean between $\sin x + \cos x$ and $\sin x - \cos x$. 11.....
- 12 Find the sum of the infinite series $3, \frac{3}{2}, \frac{3}{4}, \dots$ 12.....
- 13 Solve for x the following equation: $\sqrt{4x^2 + 5} = 2x + 1$ 13.....
- 14 What is the slope of the line whose equation is $2y = 3x + 5$? 14.....
- 15 What value of A between 90° and 180° satisfies the equation $2 \sin A - 1 = 0$? 15.....
- 16 Write the first two terms of the expansion $(2 + b)^5$ 16.....
- 17 For what value of x between 0° and 180° does the graph of $y = \cos x$ cross the x axis? 17.....
- 18 If $\tan A = \frac{1}{2}$ and $\tan B = \frac{1}{3}$, find the value of $\tan (A + B)$. 18.....
- 19 Find the radius of the circle whose center is at the origin and which passes through the point $(3, 4)$. 19.....
- 20 For what value of A between 0° and 90° will $\sin 3A$ have a maximum value? 20.....

See instructions for groups II and III on page 1.

Answer five questions from groups II and III, including at least two questions from each group.

Group II

Answer at least two questions from this group.

21 Find, correct to the nearest tenth, the roots of the equation $3x^2 = 5x + 7$ [10]

22 Solve the following pair of simultaneous equations, group your answers and check one set:

$$\begin{aligned} 2x^2 - y^2 &= 23 \\ x^2 + 2y^2 &= 34 \end{aligned} \quad [7, 2, 1]$$

23 A merchant bought some pieces of cloth for \$210. If the price had been \$1 less for each piece he could have bought 5 more pieces for the same money. Find the number of pieces purchased. [10]

24 a Draw the graph of the equation $y = x^2 - 4x - 5$ from $x = -3$ to $x = +7$ inclusive. [6]

b Draw the axis of symmetry of the curve made in answer to a. [1]

c Write the coordinates of the minimum point. [1]

d On the graph made in answer to a, draw the line which intersects the graph at the points whose abscissas are the roots of the equation $x^2 - 4x - 5 = 7$ [2]

*25 Solve the following problem graphically:

The distance between Albany and New York is 150 miles. A left Albany for New York at noon and traveled at the rate of 30 miles an hour. At the same time, B left New York for Albany. He arrived at Albany $2\frac{1}{2}$ hours after A arrived at New York. If they traveled over the same route, at what distance from Albany did they pass each other? [10]

Group III

Answer at least two questions from this group.

26 From a point on the ground the angle of elevation of a balloon is 28° . After the balloon ascends vertically a distance of 165 feet, the angle of elevation from the same point on the ground is 47° . Find the height of the balloon in the latter position. [10]

27 In triangle ABC , $c = 100$, $b = 112.5$, and angle $A = 35^\circ$. Find the value of a correct to the nearest integer. [10]

28 a Solve the equation $2 \cos^2 x + 3 \sin x = 0$ for values of x between 0° and 360° . [5]

b Prove the identity: $\cot 2A = \frac{\cot A - \tan A}{2}$ [5]

29 Derive the law of cosines for the acute triangle. [10]

* This question is based on one of the optional topics in the syllabus.