MATHEMATICS — Third Year

Wednesday, January 20, 1937 — 9.15 a. m. to 12.15 p. m., only

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Instructions

Do not open this sheet until the signal is given.

Group I

This group is to be done first and the maximum time allowed for it is one and one half hours.

If you finish group I before the signal to stop is given you may begin group II. However, it is advisable, to look your work over carefully before proceeding, since *no credit will be given any answer in group I which is not correct and in its simplest form.*

When the signal to stop is given at the close of the one and one half hour period, work on group I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

Groups II and III

Write at top of first page of answer paper to groups II and III (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in mathematics third year.

The minimum time requirement is five recitations a week for a school year after the completion of elementary algebra.

In this examination the customary lettering is used. *A, B* and *C* represent the angles of a triangle *ABC*; *a, b* and *c* represent the respective opposite sides. In a right triangle, *C* represents the right angle.

Give special attention to neatness and arrangement of work.

The use of the slide rule will be allowed for checking but all computations with tables must be shown on the answer paper.

Fill in the following lines:

Name of school..Name of pupil......................................

Detach this sheet and hand it in at the close of the one and one half hour period.

Group I

Answer all questions in this group. Each correct answer will receive $2\frac{1}{2}$ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1 Complete the formula: $\sin 2A = \ldots$

Ans............

2 Given $x^2 - 7x + 3 = 0$; by how much does the sum of the roots exceed their product?

Ans............

3 Find $\log \cos 63° \, 42'$

Ans............

4 Two sides of a triangle are 4 and 5 and the included angle is $36°$; find the area of the triangle correct to the *nearest tenth*.

Ans............

5 If $x = \sqrt[3]{71.8}$, find by logarithms the real value of x correct to the *nearest tenth*.

Ans............

6 Write the positive value of $\tan \left(\sin^{-1} \dfrac{\sqrt{3}}{2}\right)$

Ans............

7 In triangle ABC, $b = 5\sqrt{2}$, $B = 30°$, $C = 45°$; find c.

Ans............

8 Write the numerical value of $\cot (-135°)$

Ans............

9 Write the first *two* terms of the expansion $(x - \frac{1}{2})^6$

Ans............

10 Express the tangent of a positive acute angle in terms of the cosine of that angle.

Ans............

11 The roots of the equation $6x^2 - 5x + 1 = 0$ are (a) real and equal, (b) real and unequal or (c) imaginary. Which is correct, (a), (b) or (c)?

Ans............

12 Write the *seventh* term of the series $2, \frac{4}{3}, \frac{8}{9}, \ldots$

Ans............

13 Express $\dfrac{7}{\sqrt{5} - 1}$ as a fraction with a rational denominator.

Ans............

14 Find the value of $8^{\frac{2}{3}} - 3x^0 + 4^{-1}$

Ans............

15 Given $\cos A = .28$; find, without the use of tables, the value of $\sin \frac{1}{2} A$.

Ans............

16 Given the formula $t = \pi \sqrt{\dfrac{l}{g}}$; express l as a function of g and t.

Ans............

17 In triangle ABC, $a = 7$, $b = 8$, $c = 5$; find A.

Ans............

18 Find the value of x between $180°$ and $270°$ that will satisfy the equation $3 \cos^2 x = \sin^2 x$.

Ans............

19 Write the equation of the straight line whose slope is $+4$ and whose y-intercept is 2.

Ans............

20 Given $\tan x = \dfrac{1}{a}$ and $\tan y = \dfrac{1}{b}$; in terms of a and b express $\tan(x + y)$ as a fraction in its simplest form.

Ans............

Group II

Answer three questions from this group.

21 Find, correct to the *nearest tenth*, the *positive* values of x and y that satisfy the following set of equations:

$$y = 5 - x^2$$
$$y = 2x \qquad \text{[10]}$$

22 If $600 is invested at $3\frac{1}{2}\%$ interest, compounded annually, find, correct to the *nearest dollar*, the amount at the end of 20 years. $[A = P (1 + r)^n]$ [10]

23 In a two-digit number the units digit exceeds twice the tens digit by 1 ; if the digits are reversed, the new number is 1 less than twice the original number. Find the original number. [7, 3]

24 A man was employed by one company for 20 years. He saved a certain amount of money the first year and increased his annual savings by a constant amount each year. He saved $160 during his fifth year of service and $310 during his eleventh year.

   a How much did he save the first year? [5]
   b What was the total amount saved during the period of employment? [5]

25 *a* Plot the graph of the equation $y = x^2 - 4x - 2$ from $x = -1$ to $x = 5$ inclusive. [6]
   b Draw the axis of symmetry of the curve made in answer to *a*. [1]
   c Write the coordinates of the minimum point. [1]
   d With the aid of the graph made in answer to *a* write the roots of the equation
      $x^2 - 4x - 2 = -5$ [2]

Group III

Answer two questions from this group.

26 An observer in a balloon 524 feet above the ground finds the angle of depression of an object on the ground to be $27° 25'$. After ascending vertically a certain distance, he finds the angle of depression of the same object to be $42° 15'$. Find, correct to the *nearest foot,* the height of the balloon at the time of the second observation. [5, 5]

27 Two sides of a triangular garden plot are 48 feet and 62 feet. The angle included between these sides is $51° 26'$. Find, correct to the *nearest minute,* the angle opposite side 48. [3, 7]

28 *a* Derive the law of sines for an acute triangle. [5]
   b Prove the identity: $\dfrac{1 + \tan^2 A}{1 + \cot^2 A} = \dfrac{1}{\cos^2 A} - 1$ [5]

*29 An open box is made from a rectangular piece of cardboard 8 inches long and 6 inches wide by cutting out equal squares from each corner and turning up the sides. The side of the square cut out is represented by x.

   a Express the area of the bottom of the box as a function of x. [4]
   b Express the volume of the box as a function of x. [2]
   c If the volume of the box is 16 cubic inches, what is the integral value of x? [4]

  * This question is based on one of the optional topics in the syllabus.