

The University of the State of New York

262D HIGH SCHOOL EXAMINATION

MATHEMATICS — Third Year

Thursday, January 24, 1935 — 9.15 a. m. to 12.15 p. m., only

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Instructions

*Do not open this sheet until the signal is given.*

*Answer all questions in part I and five questions from part II.*

*Part I is to be done first and the maximum time to be allowed for this part is one and one half hours. Merely write the answer to each question in the space at the right; no work need be shown.*

If you finish part I before the signal to stop is given you may begin part II. However, it is advisable to look your work over carefully before proceeding to part II, since *no credit will be given any answer in part I which is not correct and reduced to its simplest form.*

When the signal to stop is given at the close of the one and one half hour period, work on part I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

In this examination the customary lettering is used.  $A$ ,  $B$  and  $C$  represent the angles of a triangle  $ABC$ ;  $a$ ,  $b$  and  $c$  represent the respective opposite sides. In a right triangle,  $C$  represents the right angle.

Give special attention to neatness and arrangement of work.

In both parts of this examination the use of the slide rule will be allowed for checking; in part II all computations with tables must be shown on the answer paper.

# MATHEMATICS — Third Year

Thursday, January 24, 1935

Fill in the following lines:

Name of school.....Name of pupil.....

Detach this sheet and hand it in at the close of the one and one half hour period.

### Part I

*Answer all questions in this part. Each question has 2½ credits assigned to it; no partial credit should be allowed. Each answer must be reduced to its simplest form.*

- 1 Express  $\frac{5\pi}{6}$  radians in degrees. Ans.....
- 2 In the right triangle  $ABC$ ,  $AC = 500$  feet,  $A = 42^\circ 40'$ ; find, correct to the nearest foot, the length of  $BC$ . Ans.....
- 3 Write in the form  $x^2 + px + q = 0$  the quadratic equation the sum of whose roots is 6 and the product of whose roots is  $-16$ . Ans.....
- 4 Combine the following terms:  $\frac{3}{\sqrt{7}} + \frac{2}{3}\sqrt{28}$  Ans.....
- 5 What is the value of the expression  $x^{-3} + 3x^0$ , when  $x = 27$ ? Ans.....
- 6 Write the first three terms of the expansion  $(1 - x^2)^7$  Ans.....
- 7 Insert three positive geometric means between 3 and 243. Ans.....
- 8 Write the equation of the straight line that passes through the origin and the point  $(2, 4)$ . Ans.....
- 9 Reduce  $\frac{\tan 2x}{\tan x}$  to its lowest terms. Ans.....
- 10 If  $\log \cos A = 9.9880 - 10$ , find the acute angle  $A$  correct to the nearest minute. Ans.....
- 11 Find the number whose logarithm is 3.2734 Ans.....
- 12 Find  $\log \sin 115^\circ 45'$  Ans.....
- 13 In the triangle  $ABC$ ,  $a = 6$  inches,  $b = 10$  inches and  $C = \sin^{-1} .26$ ; find, correct to the nearest square inch, the area of the triangle. Ans.....
- 14 Express  $\tan x$  in terms of  $\sin x$ , if  $x$  is an angle in the first quadrant. Ans.....
- 15 Find the smallest positive angle that satisfies the equation  $2 \sin^2 x + \sin x - 1 = 0$  Ans.....
- 16 Perform the indicated operations:  

$$\left( \frac{x}{x-2} - \frac{2}{2-x} \right) \div \left( \frac{x^2-4}{x} \right)$$
Ans.....
- 17 Find, correct to the nearest tenth, the value of  $x$  in the following equation:  $10^x = 20$  Ans.....
- 18 How many solutions are there for triangle  $ABC$ , if  $b = 8$ ,  $c = 12$  and  $B = 40^\circ$ ? Ans.....
- 19 Find the value of  $\cos \frac{y}{2}$ , when  $\cos y = -\frac{3}{5}$  and  $y$  is a positive angle in the second quadrant. Ans.....
- 20 Is the expression  $(a + 4)^2 = a^2 + 8(a + 2)$  an identity? Ans.....  
 [Answer Yes or No.]

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See instructions for groups II and III on page 1.

## Group II

Answer three questions from this group.

- 21 Solve the following system of equations, correctly group your answers and check
- one*
- set:

$$\begin{aligned} 2x^2 - 3y^2 + xy &= 0 \\ x - 3y &= 9 \end{aligned} \quad [7, 2, 1]$$

- 22 If \$500 is invested at 4%, interest compounded semiannually, what will the amount be at the end of 8 years? [
- $A = P(1 + \frac{r}{2})^{2n}$
- ] [10]

- 23 An automobile radiator has a capacity of 16 quarts. It is filled with a 12½% solution of alcohol and water. How many quarts of the solution must be drained off and replaced by alcohol to make a 50% solution? [10]

- 24 The sum of three integral numbers in arithmetic progression is 9. The largest number is 7 times the square of the smallest. Find the numbers. [10]

- 25 a Draw the graph of
- $y = 2x^2 + 4x + 2$
- from
- $x = -3$
- to
- $x = 2$
- inclusive. [6]

- b From the graph made in answer to a, determine the number of real roots of the equation
- $2x^2 + 4x + 2 = 0$
- [2]

- c Give the equation of the axis of symmetry for the graph of
- $y = 2x^2 + 4x + 2$
- [2]

- \*26 Solve the following set of equations for
- $x$
- ,
- $y$
- and
- $z$
- :

$$\begin{aligned} 2x - 3y - 5z &= 5 \\ 3x + 4y + 3z &= 12 \\ x - 2y - 2z &= 0 \end{aligned} \quad [10]$$

## Group III

Answer two questions from this group.

- 27 a Prove the identity:
- $\frac{\tan 2x}{2 \tan x} = \frac{\cot^2 x}{\cot^2 x - 1}$
- [4]

- b Solve for values of
- $x$
- between
- $0^\circ$
- and
- $180^\circ$
- that satisfy the equation
- $2 - 3 \sin x = \cos 2x$
- [6]

- 28 Solve the triangle
- $ABC$
- , in which
- $a = 36.73$
- ,
- $b = 42.60$
- ,
- $B = 24^\circ 30'$
- [10]

- 29 At a point 3.59 miles from one end of a lake and 5.75 miles from the other end, the lake subtends an angle of
- $78^\circ 40'$
- ; find the length of the lake correct to the nearest hundredth of a mile. [10]

\* This question is based on one of the optional topics in the syllabus.