The University of the State of New York

253D HIGH SCHOOL EXAMINATION

MATHEMATICS — Third Year

Thursday, January 28, 1932 — 9.15 a. m. to 12.15 p. m., only

Instructions

Do not open this sheet until the signal is given.

Answer all questions in part I and five questions from part II.

Part I is to be done first and the maximum time to be allowed for this part is one and one half hours. Merely write the answer to each question in the space at the right; no work need be shown.

If you finish part I before the signal to stop is given you may begin part II. However, it is advisable to look your work over carefully before proceeding to part II, since *no credit will be given any answer in part I which is not correct and reduced to its simplest form.*

When the signal to stop is given at the close of the one and one half hour period, work on part I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

In this examination the customary lettering is used. A, B and C represent the angles of a triangle ABC; a, b and c represent the respective opposite sides. In a right triangle, C represents the right angle.

Give special attention to neatness and arrangement of work.

In both parts of this examination the use of the slide rule will be allowed for checking; in part II all computations with tables must be shown on the answer paper.

MATHEMATICS — Third Year

Thursday, January 28, 1932

Fill in the following lines:

Name of school..... Name of pupil.....

Detach this sheet and hand it in at the close of the one and one half hour period.

Part I

Answer all questions in this part. Each question has 2½ credits assigned to it; no partial credit should be allowed. Each answer must be reduced to its simplest form.

1 Give the sum of the roots of the equation $3x^2 + 7x - 6 = 0$ *Ans*.....

2 Simplify $a^{\frac{3}{3}} \div \sqrt{a}$ *Ans*.....

3 Find the logarithm of .42673 *Ans*.....

4 Give the name of the curve that is the graph of the equation $4x^2 + 6y^2 = 24$ *Ans*.....

5 Write the first three terms of the expansion of $(a - b)^6$ *Ans*.....

6 Solve the following equation: $\sqrt{x^2 - 4x} = x - 3$ *Ans*.....

7 Find the *fourth* term of the series $\frac{3}{4}, \frac{1}{2}, \frac{1}{4}, \ldots$ *Ans*.....

8 Find to infinity the sum of the series $4, -\frac{1}{2}, \frac{1}{16}, \ldots$ *Ans*.....

9 Express a simple relation between the two variables shown in the table below.

p	0	1	2	3	4	5
t	2	5	8	11	14	17

Ans.....

10 Simplify $\left(\frac{1}{a+b} + \frac{1}{a-b}\right) \div \left(\frac{1}{a-b} - \frac{1}{a+b}\right)$ *Ans*.....

11 Find tan 22° 24.6′ *Ans*.....

12 Find in degrees, minutes and seconds the positive acute value of A if $\log \cos A = 9.38841 - 10$ *Ans*.....

13 If $a = 5$, $b = 8$ and $C = 60°$, where a, b and C are parts of the triangle ABC, find c. *Ans*.....

14 From the top of a cliff 500 feet in height the angle of depression of a boat is 32° 15′; find to the *nearest foot* the distance of the boat from the base of the cliff. *Ans*.....

15 Find the numerical value of $\cos 180° + \tan 45°$ *Ans*.....

16 As angle A changes from 90° to 180°, between what two values does $\sin A$ range? *Ans*.....

17 Does $\csc 60° = \frac{1}{\sin 30°}$? [Answer *Yes* or *No*.] *Ans*.....

18 What is the smallest positive value of $\cot^{-1} \sqrt{3}$? *Ans*.....

19 From the following set of equations derive a quadratic equation in y reduced to its simplest form and having 0 as its second member:

$$x^2 + y^2 = 16$$
$$x - 2y = 3$$

Ans.....

20 If $\sin A = \frac{5}{13}$ and A is in the second quadrant, find the value of $\cos A$. *Ans*.....

Write at top of first page of answer paper to part II (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in (1) elementary algebra, (2) mathematics, third year.

The minimum time requirement is five recitations a week for a school year after the completion of elementary algebra.

Part II

Answer five questions from this part, selecting three questions from group I and two from group II.

Group I

Answer three questions from this group.

21 A man can do a piece of work in 4 weeks working alone. A boy can do the same work in 10 weeks. After the man has worked half a week, he is joined by the boy. How long will it take them working together to finish the work? [7, 3]

22 A and B travel toward each other from cities 540 miles apart. A rides a motor cycle at 60 miles an hour for 4 hours. He then stops an hour for lunch and continues on his way at 70 miles an hour. Three hours after A started, B motors toward him from the other city at 40 miles an hour. How many hours after B starts do they meet and how many miles does B travel? [7, 3]

23 A square grass plot has a strip 3 feet wide taken from all sides for a walk. There is also one walk of the same width through the middle of the plot, parallel to a side. The area of the walks is 261 square feet. Find the size of the original grass plot. [7, 3]

24 *a* Plot the graph of $y = x^2 - 6x + 5$ from $x = -1$ to $x = 6$ inclusive. [6]

b From this graph determine the roots of the equation $x^2 - 6x + 5 = 0$ [1]

c What is the nature of the roots of the following equation as indicated by the graph: $x^2 - 6x + 5 = -5$? [1]

d What line is the axis of symmetry of the graph? [1]

e Give the coordinates of the turning point. [1]

*25 *a* Factor $x^3 - 3x^2 - 2x + 8$ [4]

b Solve the following set of equations for x, y and z: [6]

$$3x + 2y - z = 5$$
$$2x - 3y + 2z = 7$$
$$x - 5y - 3z = -12$$

Group II

Answer two questions from this group.

26 A man wished to know the width of a pond. He picked out three trees, A, B and C, choosing A and B on opposite edges of the pond, and made the following measurements: angle $BAC = 47° 14'$, angle $ACB = 54° 27'$, $AC = 1341$ feet. Find AB, the width of the pond. [10]

27 In the oblique triangle ABC, find angle A if $a = 671$, $b = 748$ and $c = 376$ [10]

28 *a* Solve for positive values of x less than $360°$:

$5 \sin x - 2 \cos^2 x = 1$ [5]

b Prove the following identity:

$\tan^2 x - \sin^2 x = \tan^2 x \sin^2 x$ [5]

*29 *a* Assuming the formula for $\cos (A + B)$, derive a formula for $\cos 2A$ in terms of $\sin A$. [5]

b If $\cos x = \frac{1}{4}$ and x is an angle in the first quadrant, find the value of $\cos \frac{x}{2}$ and $\tan \frac{x}{2}$. [5]

[Answer may be left in radical form.]

*This question is based on optional topics in the syllabus.