The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

TENTH YEAR MATHEMATICS

Wednesday, August 16, 1972 — 8:30 to 11:30 a.m., only

The last page of the booklet is the answer sheet, which is perforated. Fold the last page along the perfora-
tion and then, slowly and carefully, tear off the answer sheet. Now fill in the heading of your answer sheet. On
page 5, which is perforated, you will find the "Tables of Natural Trigonometric Functions," which you may
need to answer some questions in this examination. Fold this page along the perforation, and tear it off. When
you have torn off these two pages and finished the heading, you may begin the examination immediately.

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form. Write your answers in the spaces provided on the separate answer sheet.

1 The measures of the angles of a triangle are in the ratio $1:3:5$. Find the number of degrees in the measure of the *smallest* angle of the triangle.

2 Line a is parallel to line b. If transversal t meets both lines so that $\angle 1$ and $\angle 2$ are one pair of alternate interior angles and $\angle 3$ and $\angle 4$ are the other pair of alternate interior angles, find $m\angle 1 + m\angle 3$.

3 In the accompanying figure, trapezoid $ABCD$, with bases $\overline{AB}$ and $\overline{CD}$, is inscribed in circle O. If $m\widehat{AB} = 60$ and $m\widehat{CD} = 160$, find $m\widehat{AD}$.
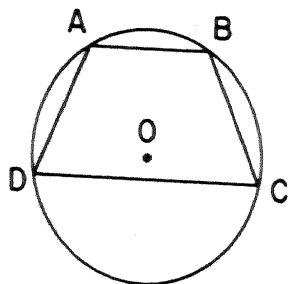
4 In $\triangle ABC$, $\overline{AB} \cong \overline{AC}$, and $m\angle B = 30$. Find $m\angle A$.

5 A triangle has sides of length 2, $\sqrt{3}$, and 1, respectively. What is the number of degrees in the measure of the *smallest* angle of the triangle?

6 Chords $\overline{AB}$ and $\overline{CD}$ intersect within a circle at point E. If $AE = 5$, $EB = 3$, and $CE = 4$, find ED.

7 The length of a side of a square is represented by $x + 2$ and the length of a side of an equilateral triangle by $2x$. If the square and the equilateral triangle have equal perimeters, find x.

8 In triangle ABC, $AB = 10$ and $m\angle B = 24$. Find to the nearest integer the length of the altitude from A to side $\overline{BC}$.

9 Two similar polygons have perimeters of 20 and 45, respectively. If the length of a side of the smaller polygon is 4, find the length of the corresponding side of the larger polygon.

10 In the figure below, $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$, $m\angle x = 40$, and $m\angle y = 80$. Find $m\angle z$.
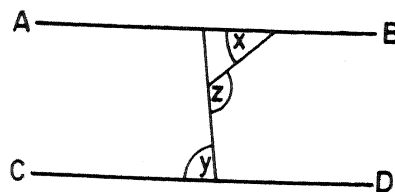
11 In the figure below, secants $\overline{PBA}$ and $\overline{PDC}$ intersect circle O. If $m\angle P = 40$ and $m\widehat{AC}:m\widehat{BD} = 2:1$, find the numerical value of $m\widehat{BD}$.
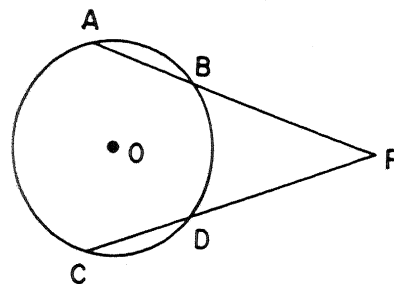

12 Express in radical form the length of the line segment joining the points $(3,4)$ and $(7,2)$.

13 Find the coordinates of the midpoint of the line segment joining the points whose coordinates are $(3,5)$ and $(7,11)$.

14 The length of chord $\overline{AB}$ in circle with center at O is 6. If the radius of the circle is 5, find the distance from O to $\overline{AB}$.

15 In the figure below, circle O has a radius of 4. If $m\angle AOB = 90$, express, in terms of π, the length of minor arc AB.
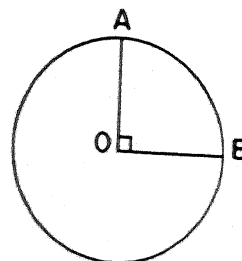
16 How many degrees are in the sum of the measures of the interior angles of a polygon with 12 sides?

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Math. 10–Aug. '72

[2]

Math.

Directions (17–29): For *each* statement or question, write on the separate answer sheet the *numeral* preceding the word or expression that, of those given, best completes the statement or answers the question.

17 If x units are added to the length of the radius of a circle, what is the number of units by which the circumference of the circle is increased?

(1) x (3) 2π
(2) 2   (4) $2\pi x$

18 To prove that an angle is an obtuse angle, it is sufficient to prove that the angle

(1) is not a right angle
(2) is not a straight angle
(3) is neither a right nor a straight angle
(4) measures greater than 90° and less than 180°

19 If the base of an isosceles triangle is 10, then the length of one of the equal sides could be

(1) 6   (3) 3
(2) 5   (4) 4

20 Given triangle ABC and triangle DEF such that $\overline{BC} \cong \overline{BA}$, $\overline{AB} \cong \overline{DF}$, $\overline{DE} \cong \overline{FE}$, and $\angle A \cong \angle E$. Which statement must be true?

(1) $\triangle ABC \cong \triangle DEF$
(2) $\triangle ABC \sim \triangle DEF$
(3) $\triangle ABC$ and $\triangle DEF$ are isosceles.
(4) $\triangle ABC$ and $\triangle DEF$ are equal in area.

21 Two right triangles must be congruent if

(1) the altitude to the hypotenuse of one triangle is congruent to the altitude to the hypotenuse of the other triangle
(2) the hypotenuse of one triangle is congruent to the hypotenuse of the other triangle
(3) the two legs of one triangle are, respectively, congruent to the two legs of the other triangle
(4) an acute angle of one triangle is congruent to an acute angle of the other triangle

22 In triangle ABC, if $m\angle A = 50$ and $m\angle B = 60$, then

(1) $m\angle C > 90$ (3) $AB < BC$
(2) $BC > AC$ (4) $AB > AC$

23 If two circles with radii 4 and 2, respectively, are internally tangent, then the distance between their centers is

(1) 0   (3) 3
(2) 2   (4) 6

24 Two regular polygons with the same number of sides are always

(1) equal in area   (3) similar
(2) congruent   (4) coincident

25 In right triangle ABC with legs a and b and hypotenuse c, the value of $(c - a)(c + a)$ is always the same as the value of

(1) $a^2 + b^2$ (3) b^2
(2) $a^2 + c^2$ (4) c^2

26 The area of a rhombus is 36. If the length of one diagonal is 9, the length of the other diagonal is

(1) 18   (3) 8
(2) 9   (4) 4

27 The area of a square is 10. The length of a side of that square is

(1) exactly 2.5
(2) between 3 and 4
(3) between 4 and 5
(4) exactly 5

28 In the accompanying diagram the total number of points which are ¼ inch from C and also equidistant from $\overleftrightarrow{AD}$ and $\overleftrightarrow{BE}$ is
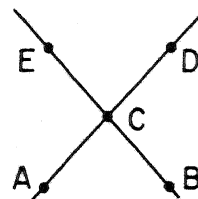
(1) 1   (3) 3
(2) 2   (4) 4

29 If two circles in the same plane have exactly two points in common, what is the greatest number of common tangent lines that can be drawn to these circles?

(1) 1   (3) 3
(2) 2   (4) 4

Directions (30): Leave all construction lines on the answer sheet.

30 *On the answer sheet*, construct the line through A perpendicular to $\overleftrightarrow{BC}$.

Answers to the following questions are to be written on paper provided by the school.

Part II

Answer four questions from this part. Show all work unless otherwise directed.

31 Prove *either* a or b, but *not* both: [10]

a An angle formed by two secants intersecting outside the circle is measured by one-half the difference of the intercepted arcs.

OR

b If in a right triangle the altitude is drawn upon the hypotenuse,
   (1) the two triangles thus formed are similar to the given triangle and similar to each other,
   (2) each leg of the given triangle is the mean proportional between the hypotenuse and the projection of that leg on the hypotenuse.

32 In the diagram below, triangle ABC is inscribed in circle O. $\overleftrightarrow{PA}$ is tangent to circle O at point A. $\overleftrightarrow{RS}$ is parallel to $\overleftrightarrow{PA}$ and intersects $\overline{AB}$ at R and $\overline{AC}$ at S.
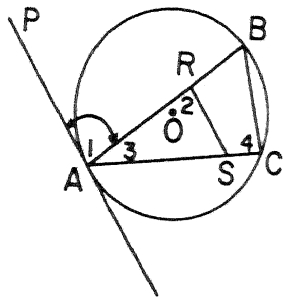

Prove: $AR \times AB = AS \times AC$ [10]

33 In the figure below, base $\overline{AB}$ of $\square ABDE$ is extended to C and $\overline{CD}$ is drawn. $\overline{CD} \cong \overline{BD}$, $m\angle A = 40$, $ED = 10$, $BD = 8$.
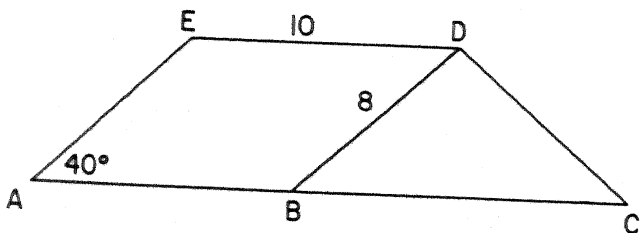
a Find, to the *nearest integer*, the length of the altitude of $\triangle DBC$ drawn from D to side $\overline{BC}$. [3]
b Find, to the *nearest integer*, the length of $\overline{BC}$. [3]
c Using your results from parts a and b, find the area of $\triangle BDC$. [2]
d Using your result from part a, find the area of $\square ABDE$. [2]

34 *Given*: Quadrilateral $ABCD$ with points E and F on diagonal $\overline{AC}$, $\overline{AF} \cong \overline{EC}$, $\overline{BF} \parallel \overline{ED}$, and $\overline{BF} \cong \overline{ED}$.
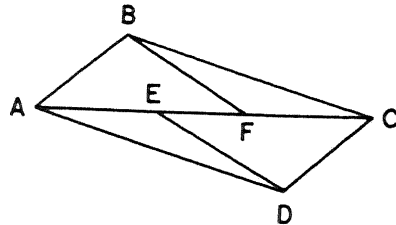
Prove: $ABCD$ is a parallelogram [10]

35 The points A, B, and C have coordinates $(-3,5)$, $(1,5)$, and $(1,-3)$, respectively.
a Write an equation of the locus of points equidistant from A and B. [2]
b Write an equation of the locus of points equidistant from B and C. [2]
c Find the coordinates of the center of the circle which passes through A, B, and C. [2]
d Express in radical form the length of the radius of the circle which passes through A, B, and C. [2]
e Find the area of $\triangle ABC$. [2]

36 In the diagram below, $\triangle ABC$, $\overline{BE}$ bisects $\angle B$, $\overline{AE}$ bisects $\angle A$, and $CA > CB$.
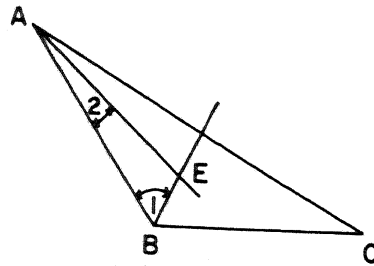
Prove: $m\angle 1 > m\angle 2$ [10]

*37 Parallelogram $ABCD$ has vertex A at $(-5,0)$, vertex B at $(0,7)$, vertex C at (x,y), and vertex D at $(4,0)$.
a Express the slope of $\overline{BC}$ in terms of x and y. [3]
b Express the slope of $\overline{AD}$ numerically. [2]
c Use the results of parts a and b to solve for y. [3]
d Find the value of x. [2]

* This question is based on an optional topic in the syllabus.

Math

Tables of Natural Trigonometric Functions
(For use with 9th and 10th Year Mathematics Regents Examinations)

Angle	Sine	Cosine	Tangent	Angle	Sine	Cosine	Tangent
1°	.0175	.9998	.0175	46°	.7193	.6947	1.0355
2°	.0349	.9994	.0349	47°	.7314	.6820	1.0724
3°	.0523	.9986	.0524	48°	.7431	.6691	1.1106
4°	.0698	.9976	.0699	49°	.7547	.6561	1.1504
5°	.0872	.9962	.0875	50°	.7660	.6428	1.1918
6°	.1045	.9945	.1051	51°	.7771	.6293	1.2349
7°	.1219	.9925	.1228	52°	.7880	.6157	1.2799
8°	.1392	.9903	.1405	53°	.7986	.6018	1.3270
9°	.1564	.9877	.1584	54°	.8090	.5878	1.3764
10°	.1736	.9848	.1763	55°	.8192	.5736	1.4281
11°	.1908	.9816	.1944	56°	.8290	.5592	1.4826
12°	.2079	.9781	.2126	57°	.8387	.5446	1.5399
13°	.2250	.9744	.2309	58°	.8480	.5299	1.6003
14°	.2419	.9703	.2493	59°	.8572	.5150	1.6643
15°	.2588	.9659	.2679	60°	.8660	.5000	1.7321
16°	.2756	.9613	.2867	61°	.8746	.4848	1.8040
17°	.2924	.9563	.3057	62°	.8829	.4695	1.8807
18°	.3090	.9511	.3249	63°	.8910	.4540	1.9626
19°	.3256	.9455	.3443	64°	.8988	.4384	2.0503
20°	.3420	.9397	.3640	65°	.9063	.4226	2.1445
21°	.3584	.9336	.3839	66°	.9135	.4067	2.2460
22°	.3746	.9272	.4040	67°	.9205	.3907	2.3559
23°	.3907	.9205	.4245	68°	.9272	.3746	2.4751
24°	.4067	.9135	.4452	69°	.9336	.3584	2.6051
25°	.4226	.9063	.4663	70°	.9397	.3420	2.7475
26°	.4384	.8988	.4877	71°	.9455	.3256	2.9042
27°	.4540	.8910	.5095	72°	.9511	.3090	3.0777
28°	.4695	.8829	.5317	73°	.9563	.2924	3.2709
29°	.4848	.8746	.5543	74°	.9613	.2756	3.4874
30°	.5000	.8660	.5774	75°	.9659	.2588	3.7321
31°	.5150	.8572	.6009	76°	.9703	.2419	4.0108
32°	.5299	.8480	.6249	77°	.9744	.2250	4.3315
33°	.5446	.8387	.6494	78°	.9781	.2079	4.7046
34°	.5592	.8290	.6745	79°	.9816	.1908	5.1446
35°	.5736	.8192	.7002	80°	.9848	.1736	5.6713
36°	.5878	.8090	.7265	81°	.9877	.1564	6.3138
37°	.6018	.7986	.7536	82°	.9903	.1392	7.1154
38°	.6157	.7880	.7813	83°	.9925	.1219	8.1443
39°	.6293	.7771	.8098	84°	.9945	.1045	9.5144
40°	.6428	.7660	.8391	85°	.9962	.0872	11.4301
41°	.6561	.7547	.8693	86°	.9976	.0698	14.3007
42°	.6691	.7431	.9004	87°	.9986	.0523	19.0811
43°	.6820	.7314	.9325	88°	.9994	.0349	28.6363
44°	.6947	.7193	.9657	89°	.9998	.0175	57.2900
45°	.7071	.7071	1.0000	90°	1.0000	.0000	

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The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

TENTH YEAR MATHEMATICS

Wednesday, August 16, 1972 — 8:30 to 11:30 a.m., only

ANSWER SHEET

Pupil...Teacher...

School...

Name and author of textbook used...

Your answers to Part I should be recorded on this answer sheet.

Part I
Answer all questions in this part.

1............................	11............................	21............................
2............................	12............................	22............................
3............................	13............................	23............................
4............................	14............................	24............................
5............................	15............................	25............................
6............................	16............................	26............................
7............................	17............................	27............................
8............................	18............................	28............................
9............................	19............................	29............................
10............................	20............................	

Answer question 30 on the back of this page.

A •

B ———————•—————————•——— C

TENTH YEAR MATHEMATICS

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ERRATA SHEET

At the start of the examination in Tenth Year Mathematics, please inform your pupils, either orally or by writing the correction on the chalk board, that in line 2 of question 34 the line over the first set of letters " ED " should extend over *both* letters, ie. $\overline{ED}$.

FOR TEACHERS ONLY

10

SCORING KEY

TENTH YEAR MATHEMATICS

Wednesday, August 16, 1972 — 8:30 to 11:30 a.m., only

Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 17–29, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3, or 4.

(1) 20	(11) 80	(21) 3
(2) 180	(12) $2\sqrt{5}$	(22) 4
(3) 70	(13) (5,8)	(23) 2
(4) 120	(14) 4	(24) 3
(5) 30	(15) 2π	(25) 3
(6) $15/4$	(16) 1,800	(26) 3
(7) 4	(17) 4	(27) 2
(8) 4	(18) 4	(28) 4
(9) 9	(19) 1	(29) 2
(10) 120	(20) 3	

[OVER]

Part II

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics*. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(33)   *a*   5   [3]

      b   12   [3]

      c   30   [2]

      d   50   [2]

(37)   *a* $\dfrac{y-7}{x}$ [3]

      b   0   [2]

      c   7   [3]

      d   9   [2]

(35)   *a* $x = -1$ [2]

      b $y = 1$ [2]

      c $(-1,1)$ [2]

      d $2\sqrt{5}$ [2]

      e   16   [2]

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you

DO YOU KNOW . . .

Who writes the questions used on Regents examinations?

1 the members of the Board of Regents
2 the subject supervisors in the State Education Department
3 college professors in the various disciplines
4 classroom teachers from schools throughout New York State

The correct answer is 4. Last year more than 400 classroom teachers were involved in the preparation of Regents examination questions, and many other teachers served on the committee that assembled the examinations.