The University of the State of New York

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REGENTS HIGH SCHOOL EXAMINATION

TENTH YEAR MATHEMATICS

Tuesday, August 16, 1966 — 8:30 to 11:30 a.m., only

The last page of the booklet is the answer sheet, which is perforated. Fold the last page along the perforation and then, slowly and carefully, tear off the answer sheet. Now fill in the heading of your answer sheet. When you have finished the heading, you may begin the examination immediately.

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers in the spaces provided on the separate answer sheet.

1 In triangle ABC, angle C is a right angle, angle $A = 60°$, and side $AC = 4$. Find the length of side AB.

2 Find the number of degrees in each interior angle of a regular polygon having 12 sides.

3 Diagonal AC of parallelogram $ABCD$ is 16 inches long. Point M is the midpoint of AD and point N is the midpoint of CD. Find the number of inches in the length of segment MN.

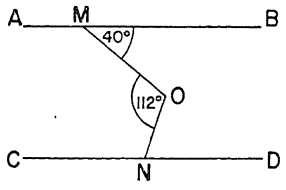
4 The area of a rhombus is 42. The length of one diagonal is 6. Find the length of the other diagonal.

5 In triangle ABC, angle $C = 60°$ and $\angle C < \angle B$. Name the longest side of the triangle.

6 The length of altitude CD to hypotenuse AB of right triangle ABC is $\sqrt{6}$. If the length of segment AD is 3, find the length of segment DB.

7 Two consecutive angles of a parallelogram are represented by $(x - 14)°$ and $(3x - 10)°$. Find the value of x.

8 In the accompanying figure, $AB \parallel CD$. If angle $BMO = 40°$ and angle $MON = 112°$, find the number of degrees in angle DNO.



9 The coordinates of the end points of a line segment are A (6,3) and C (—2,—3). Find the length of AC.

10 In circle O, chords AB and CD intersect at E. If $AE = x$, $EB = x - 1$, $CE = x + 2$, and $ED = x - 2$, find the value of x.

11 In $\triangle ABC$, a line parallel to AB intersects AC at D and BC at E. If $AB = 15$, $DE = 6$, and $DC = 2$, find AC.

12 The end points of a line segment are A (3,4) and B (8,10). Find the coordinates of the midpoint of line segment AB.

13 An angle of 40° is formed by a tangent and a secant drawn to a circle from an external point. If the larger arc intercepted by this angle is 140°, find the number of degrees in the measure of the smaller intercepted arc.

14 Regular pentagon $ABCDE$ is inscribed in a circle. If diagonals AC and EC are drawn, find in degrees the measure of angle ACE.

15 Express in terms of π the area of a circle whose diameter is 18.

16 In triangle ABC, AD is a median. What is the ratio of the area of triangle ABD to the area of triangle ABC?

17 The radius of a circle is 6. The length of chord AB of this circle is also 6. Find in radical form the distance from the center to the chord AB.

18 Express in terms of π the length of an arc of 45° in a circle of radius 12.

19 In triangle ABC, angle $A = 36°$ and $AB = AC$. If the bisector of angle C meets AB in D and $DC = 3$, find the length of AD.

20 Tangent PA is drawn to a circle at point A. The radius of the circle is 6 and $PA = 8$. Find the distance from P to the center of the circle.

21 Find to the *nearest degree* the angle of elevation of the top of a 20-foot vertical pole from a point on level ground 30 feet from the foot of the pole.

22 Write an equation of the locus of points whose abscissas are 3 more than their ordinates.

23 The perimeter of a regular polygon is 48 and its apothem is 6. Find the area of the polygon.

Directions (24–28) : Write in the space provided on the separate answer sheet the *number* preceding the expression that best completes the statement or answers the question.

24 What is the total number of points which are 3 inches from a given line and also 3 inches from a given point on this line?

(1) 1 (3) 3
(2) 2 (4) 4

25 If each side of a triangle is multiplied by 3, how would the perimeter and the area of the triangle be changed?

(1) The perimeter and the area are both multiplied by 3.
(2) The perimeter and the area are both multiplied by 9.
(3) The perimeter is multiplied by 3 and the area is multiplied by 9.
(4) The perimeter is multiplied by 9 and the area is multiplied by 3.

26 If two circles are internally tangent to each other, what is the greatest number of common tangents that can be drawn to them?

(1) 1 (3) 3
(2) 2 (4) 0

27 Which statement is the inverse of the statement "If John studies, he does successful work"?

(1) If John does not do successful work, he has not studied.
(2) John does successful work if he studies.
(3) If John does not study, he does not do successful work.
(4) If John does successful work, then he has studied.

28 A quadrilateral is inscribed in a circle. If one angle of the quadrilateral is 50°, the opposite angle must be

(1) 310° (3) 50°
(2) 130° (4) 40°

Directions (29–30) : Leave all construction lines on the answer sheet.

29 *On the answer sheet,* construct on $A'B'$ triangle $A'B'C'$ similar to triangle ABC.

30 *On the answer sheet,* construct the locus of points equidistant from points A and B.

Answers to the following questions are to be written on paper supplied by the school.

Part II

Answer four questions from this part. Show all work unless otherwise directed.

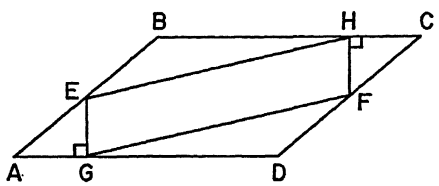
31 Prove *either a* or *b* but *not* both: [10]

a Two right triangles are congruent if the hypotenuse and a leg of one are equal respectively to the hypotenuse and a leg of the other.

OR

b The area of a trapezoid is equal to one-half the product of the altitude and the sum of the bases.

32 In the accompanying figure, $ABCD$ is a parallelogram. E is the midpoint of AB and F is the midpoint of CD. EG is $\perp$ AD and FH is $\perp$ BC. EH and GF are drawn.



Prove:

a $AG = HC$ [6]

b $EGFH$ is a parallelogram [4]

33 In right triangle ABC, AB is the hypotenuse. A perpendicular to AB at A meets BC extended at E. A perpendicular to AB at B meets AC extended at D.

Prove: $\frac{EA}{CB} = \frac{AB}{CD}$ [10]

34 Point A lies outside a circle whose center is at O. Line segment OA is 37 inches long. From A, a tangent AB is drawn to the circle. If this tangent line forms an angle of 27° with OA, find to the *nearest inch* the length of the

a radius of the circle [6]

b tangent to the circle [4]

35 The vertices of quadrilateral $MNPR$ are M $(a,2b)$, N $(4a,4b)$, P $(9a,4b)$, and R $(6a,2b)$.

(1) Using coordinate geometry, express in terms of a and b the length of each side of quadrilateral $MNPR$. [6]

(2) Using the results found in answer to (1), classify the quadrilateral $MNPR$. [1]

(3) If $a = 1$ and $b = 2$, find the lengths of sides MN and MR. [2]

(4) [After the numeral 4 on your answer paper write the *number* of the expression which best completes the statement.] [1]

It follows from (1), (2), and (3) that quadrilateral $MNPR$ must be a

(1) rectangle (2) rhombus (3) square

36 In isosceles trapezoid $ABCD$, base AB is 4 inches longer than base DC. The altitude is equal to the shorter base. The area of the trapezoid is 24 square inches.

a Find the number of inches in the length of each base of the trapezoid. [6]

b If the nonparallel sides, AD and BC, are extended to meet at point E, find the area of triangle CED. [4]

*37 The coordinates of the vertices of $\triangle ABC$ are A $(-2,5)$, B $(2,1)$, and C $(6,7)$. The midpoint of AB is M and the midpoint of BC is N.

a Find the coordinates of M and N. [3]

b Find the slope of the line through points M and N. [2]

c Using coordinate geometry, show that MN is parallel to AC, and write a reason for this conclusion. [3]

d Write an equation of the line MN. [2]

* This question is based on an optional topic in the syllabus.

FOR TEACHERS ONLY

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SCORING KEY

TENTH YEAR MATHEMATICS

Tuesday, August 16, 1966 — 8:30 to 11:30 a.m., only

Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 24–28, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3, or 4.

(1) 8

(2) 150

(3) 8

(4) 14

(5) AC or b

(6) 2

(7) 51

(8) 72

(9) 10

(10) 4

(11) 5

(12) $(5\frac{1}{2},7)$

(13) 60

(14) 36

(15) 81π

(16) 1:2

(17) $3\sqrt{3}$ *or* $\sqrt{27}$

(18) 3π

(19) 3

(20) 10

(21) 34

(22) $x = y + 3$ (*or* equivalent)

(23) 144

(24) 2

(25) 3

(26) 1

(27) 3

(28) 2

[OVER]

Part II

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics*. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(34) *a* 17 [6]
b 33 [4]

(35) (1) $MN = \sqrt{9a^2 + 4b^2}$
$NP = 5a$
$PR = \sqrt{9a^2 + 4b^2}$
$RM = 5a$ [6]

(2) $MNPR$ is a parallelogram. [1]

(3) $MN = 5$
$MR = 5$ [2]

(4) (2) [1]

(36) *a* 4 and 8 [6]
b 8 [4]

(37) *a* (0,3) and (4,4) [3]
b $\frac{1}{4}$ [2]
c Lines MN and AC are parallel because their slopes are equal. [3]
d $y = \frac{1}{4}x + 3$ [2]