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The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

TENTH YEAR MATHEMATICS

Monday, August 17, 1964 — 8:30 to 11:30 a.m., only

The last page of the booklet is the answer sheet, which is perforated. Fold the last page along the perforation and then, slowly and carefully, tear off the answer sheet. Now fill in the heading of your answer sheet. When you have finished the heading, you may begin the examination immediately.

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers in the spaces provided on the separate answer sheet.

1 The diagonal of a rectangle is 17 and its altitude is 8. Find the length of the base.

2 From an external point P, tangents PA and PB are drawn to a circle. If $PA = 5x - 2$ and $PB = 3x + 4$, find the value of x.

3 The area of trapezoid $ABCD$ is 60 square inches. The altitude is 4 inches and base AB is 17 inches long. Find the number of inches in the length of base CD.

4 Each interior angle of a polygon contains 150°. Find the number of sides of the polygon.

5 The ratio of the radii of two regular polygons of the same number of sides is 1 : 4. The area of the smaller polygon is 4 square feet. Write the number of square feet in the area of the larger polygon.

6 Find the length of the line segment joining the points whose coordinates are (—3,2) and (6,—1). [Answer should be left in radical form.]

7 Write the coordinates of the midpoint of the line segment joining the points whose coordinates are $(3a,-b)$ and (a,b).

8 The sum of the complement and the supplement of a certain angle is 168°. Find the number of degrees in the angle.

9 The radius of a circle is 9. Express in terms of π the area of a sector of 10°.

10 From point P outside a circle, two secants PAB and PCD are drawn, intercepting larger arc BD containing 105° and smaller arc AC. Angle P contains 34°. Find the number of degrees in arc AC.

11 A vertical pole 8 feet high casts a shadow 11 feet long on level ground. Find to the *nearest degree* the angle of elevation of the sun.

12 In a circle, chord AB bisects chord CD at E. The lengths of AE and EB are 3 inches and 27 inches, respectively. How many inches long is chord CD?

13 One angle of a rhombus contains 120°, and the shorter diagonal is 8 inches long. Find in radical form the number of square inches in the area of the rhombus.

14 Find in terms of π the circumference of a circle whose area is $\frac{25}{4}\pi$.

15 The area of a regular polygon is 121 and its perimeter is 44. Find the length of its apothem.

16 The area of a square is 32. Find the length of a diagonal.

17 Quadrilateral $ABCD$ is inscribed in a circle. Angle A contains 58°. How many degrees are in angle C?

18 A tangent and a secant are drawn to a circle from an external point. The secant is 18 inches long, and the external segment of the secant is 2 inches long. Find the number of inches in the length of the tangent.

19 In triangle ABC, D is a point on AB between A and B; and E is a point on AC between A and C so that DE is parallel to BC. If $AD = 2$, $DB = 3$ and $BC = 10$, find the length of DE.

20 In right triangle ABC, CD is the altitude drawn to the hypotenuse AB. If $AD = 5$ and $AC = 6$, find the length of AB.

21 Write an equation of the locus of points whose abscissas are 5 less than their ordinates.

22 In regular pentagon $ABCDE$, diagonal AD is drawn. Find the number of degrees in angle BAD.

Directions (23–28): Write in the space provided on the separate answer sheet the *number* preceding the expression that best completes *each* statement or answers *each* question.

23 The point P is 2 inches from the line AB. The total number of points at a distance of 4 inches from line AB and also at a distance of 6 inches from P is

(1) 1 (2) 2 (3) 3 (4) 4

24 In parallelogram $ABCD$, the bisector of angle A intersects side CD at E. DE must equal

(1) AE
(2) EC
(3) AD
(4) the distance of E from AB

25 The lengths of two sides of a triangle are 5 and 6. The third side may be

(1) 1
(2) $\sqrt{5}$
(3) 11
(4) $\sqrt{130}$

26 Which statement is an example of a correct definition?

(1) A central angle of a circle is an angle formed by two of its radii.
(2) An acute triangle is a triangle in which one angle is acute.
(3) A parallelogram is a figure with opposite sides parallel.
(4) A regular polygon is a polygon which is equiangular.

27 In triangle ABC, angle C is a right angle. R, S and T are the midpoints of AB, BC and CA, respectively. Then

(1) $CR > ST$
(2) $CR = ST$
(3) $CR < BR$
(4) $CR > BR$

28 Given the statement " If the opposite sides of a quadrilateral are equal, the quadrilateral is a parallelogram." Which is an inverse of this statement?

(1) If the opposite sides of a quadrilateral are parallel, the quadrilateral is a parallelogram.
(2) If a quadrilateral is not a parallelogram, the opposite sides of the quadrilateral are not equal.
(3) If a quadrilateral is a parallelogram, the opposite sides are equal.
(4) If the opposite sides of a quadrilateral are not equal, the quadrilateral is not a parallelogram.

Directions (29–30): Leave all construction lines on the answer sheet.

29 Given triangle ABC and line segment DE on the answer sheet. Construct triangle FDE similar to triangle ABC so that BC and DE are corresponding sides.

30 Given rectangle $ABCD$ on the answer sheet. Find, by construction, the point on DC which is equidistant from AB and AD.

Answers to the following questions are to be written on paper furnished by the school.

Part II

Answer four questions from this part. Show all work unless otherwise directed.

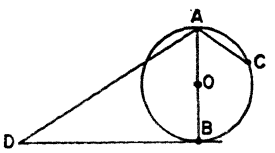
31 Prove *either a* or *b* but *not* both: [10]

a An angle formed by two chords intersecting inside the circle is measured by one-half the sum of the intercepted arcs.

OR

b The square of the hypotenuse of a right triangle is equal to the sum of the squares of the legs.

32 In the figure shown, AB is a diameter of circle O. DB is tangent to the circle at B, and AB bisects angle DAC.



Prove: $AC \times AD = (AB)^2$ [10]

33 Trapezoid $ABCD$, with AB the larger base, is inscribed in a circle. Sides AD and BC are extended to meet at P. Diagonals AC and BD meet at R. Arc CD contains 60 degrees and BD is perpendicular to AC.

a Find the number of degrees in arc AB, arc AD and arc CB. [2, 2, 1]

b Find the number of degrees in angle P. [2]

c If the length of CB is 8, find the area of triangle CRB. [Answer may be left in radical form.] [3]

34 The perimeter of a regular polygon of 9 sides is 126.

a Find to the ***nearest tenth*** the length of the apothem of the polygon. [5]

b Using the result found in part *a*, find to the ***nearest integer*** the number of square units in the area of the polygon. [2]

c Find to the ***nearest integer*** the length of the radius of the circle which can be circumscribed about the regular polygon. [3]

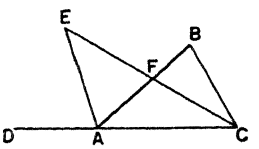
35 The coordinates of the vertices of parallelogram $ABCD$ are A $(-1,-3)$, B $(4,2)$, C $(3,9)$ and D (a,b).

a Find the coordinates of the midpoint of line segment AC. [2]

b Using the results found in part *a*, determine the coordinates of point D. [4]

c Show that parallelogram $ABCD$ is a rhombus. [4]

36 In the figure shown, bisector AE of exterior angle DAB of triangle ABC meets bisector CE of angle ACB at E.



a Prove: $\angle EAB > \angle BCE$ [5]

b In triangles EAF and BCF, prove: $\angle E < \angle B$ [5]

*37 The vertices of quadrilateral $ABCD$ are A $(0,0)$, B $(0,-3)$, C $(7,4)$ and D $(4,4)$.

a Using graph paper, draw quadrilateral $ABCD$. [1]

b Write an equation of the line passing through the points A and D. [3]

c Write an equation of the line passing through the points C and D. [2]

d Select the correct choice from the following: [2]

(1) $ABCD$ is a parallelogram.
(2) $ABCD$ is a trapezoid with unequal nonparallel sides.
(3) $ABCD$ is an isosceles trapezoid.
(4) $ABCD$ is a rhombus.

e Find the number of degrees in angle DCB. [2]

*This question is based on an optional topic in the syllabus.

FOR TEACHERS ONLY

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SCORING KEY
TENTH YEAR MATHEMATICS

Monday, August 17, 1964 — 8:30 to 11:30 a.m., only

Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 23–28, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3 or 4.

(1) 15

(2) 3

(3) 13

(4) 12

(5) 64

(6) $\sqrt{90}$ *or* $3\sqrt{10}$

(7) $(2a, 0)$

(8) 51

(9) $\frac{9}{4}\pi$

(10) 37

(11) 36

(12) 18

(13) $32\sqrt{3}$

(14) 5π

(15) $\frac{11}{2}$

(16) 8

(17) 122

(18) 6

(19) 4

(20) $7\frac{1}{5}$

(21) $x = y - 5$ *or* $y = x + 5$

(22) 72

(23) 3

(24) 3

(25) 2

(26) 1

(27) 2

(28) 4

Part II

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics*. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(33) *a* 120, 90, 90 [2, 2, 1]
b 30 [2]
c $8\sqrt{3}$ [3]

(34) *a* 19.2 [5]
b 1210 [2]
c 20 [3]

(35) *a* (1,3) [2]
b (—2,4) [4]

*(37) *b* $y = x$ [3]
c $y = 4$ [2]
d 3 [2]
e 45 [2]