The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

TENTH YEAR MATHEMATICS

Monday, August 19, 1963 — 8:30 to 11:30 a.m., only

Name of pupil..... Name of school.....

Name and author of textbook used.....

Name of teacher.....

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed.

1 The area of a square is 81. Express in radical form the length of a diagonal. 1.....

2 Find the coordinates of the midpoint of the line segment joining the points whose coordinates are (—2,7) and (4,—15). 2.....

3 In a circle whose radius is 10 inches, a chord is 6 inches from the center. Find the number of inches in the length of the chord. 3.....

4 Express in radical form the area of an equilateral triangle whose side is 6. 4.....

5 A tangent and a secant are drawn from an external point to a circle. The length of the whole secant is $4x$ and the external segment is x. The length of the tangent is 4. Find the numerical value of x. 5.....

6 The length of the shorter side of a rectangle is 3 and the diagonal is 8. Find to the *nearest degree* the angle that the diagonal makes with the longer side. 6.....

7 The perimeters of two similar rectangles are 20 and 30. The area of the smaller rectangle is 24. Find the area of the larger rectangle. 7.....

8 The bases of an isosceles trapezoid are 7 and 13 and each base angle is 45 degrees. Find the area of the trapezoid. 8.....

9 In a circle the length of an arc of 40° is 2π. What is the length of the radius? 9.....

10 In triangle ABC, a line parallel to AB intersects AC at D and BC at E. If $AC = 12$, $DC = 8$ and $AB = 18$, find DE. 10.....

11 The length of a side of a regular polygon of n sides is 5. If the apothem is a, express the area in terms of a and n. 11.....

12 The altitude is drawn to the hypotenuse of a right triangle. If the segments of the hypotenuse cut off by the altitude are 9 and 16, what is the length of the altitude? 12.....

13 Express in terms of π the area of a circle whose *diameter* is 18. 13.....

14 The coordinates of two opposite vertices of quadrilateral $ABCD$ are A (5,8) and C (—3,2). Find the length of diagonal AC. 14.....

[OVER]

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15 From a point P outside circle O, two tangents are drawn meeting the circle at points A and B. The ratio of major arc AB to minor arc AB is 7 : 2. Find the number of degrees in angle P. 15.....

16 Two sides of a triangle are 10 and 12. The angle included between them is 30°. Find the area of the triangle. 16.....

17 In $\triangle ABC$, $\angle C$ is 25° and BC is 13 inches. If the exterior angle at B is 50°, find the number of inches in side AB. 17.....

18 Chords AB and CD of a circle intersect at E. If $AE = 5$, $EB = 6$, $CE = x$ and $ED = 12$, find the value of x. 18.....

19 Write an equation of the locus of points equidistant from (—1,3) and (5,3). 19.....

20 The area of a rhombus is 24. The length of one diagonal is 8. What is the length of the other diagonal? 20.....

21 Triangle ABC is inscribed in circle O. Angle ABC contains 38°. At point A a tangent to the circle is drawn. How many degrees are in the acute angle that this tangent forms with line AC? 21.....

Directions (22–27) : Write on the line at the right of *each* of the following the *number* preceding the expression that best completes the statement or answers the question.

22 Two sides of a triangle are 3 and 7. The third side may be
(1) 9
(2) 10
(3) 3
(4) 4
22.....

23 It is possible for the number of degrees in an exterior angle of a regular polygon to equal
(1) 80°
(2) 70°
(3) 50°
(4) 10°
23.....

24 A is a point on a line. The total number of points which are at a distance of 4 units from A and also at a distance of 3 units from the given line is
(1) 1
(2) 2
(3) 3
(4) 4
24.....

25 Consider the statement " Parallel lines are lines which lie in the same plane and do not intersect however far they are extended." This statement is an example of a
(1) theorem
(2) definition
(3) postulate
(4) contradiction
25.....

26 The center of the circle that can be inscribed in a scalene triangle is the point of intersection of the
(1) medians
(2) altitudes
(3) angle bisectors
(4) perpendicular bisectors of the sides
26.....

27 Consider these statements:

a The square of the hypotenuse of a right triangle is equal to the sum of the squares of the other two sides.

b If the square of one side of a triangle is equal to the sum of the squares of the other two sides, the triangle is a right triangle.

Which of the following is true?
(1) *a* is the same as *b*
(2) *a* is the converse of *b*
(3) *a* is the inverse of *b*
(4) *a* is the converse of the inverse of *b*
27.....

Directions (28–29): If the blank space in each statement below is replaced by the word *always*, *sometimes* (but not always) or *never*, the resulting statement will be true. Select the word that will correctly complete *each* statement and write this word on the line at the right.

28 If a diagonal of a parallelogram bisects an angle of the parallelogram, the parallelogram is ... equilateral. 28.....

29 In $\triangle ABC$, $AB = BC$. D is a point on AC between A and C. AB is ... less than DB. 29.....

Directions (30): Leave all construction lines on the paper.

30 Divide line segment AB into *three* equal parts.

A ——————————————— B

Part II

Answer four questions from this part. Show all work unless otherwise directed.

31 Prove *either* *a* or *b* but *not* both:

a Two right triangles are congruent if the hypotenuse and a leg of one are equal to the corresponding parts of the other. [10]

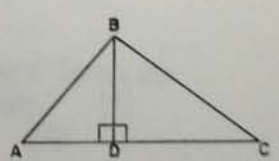
OR

b If in a right triangle the altitude is drawn upon the hypotenuse,

(1) the two triangles thus formed are similar to the given triangle and similar to each other [7] and

(2) each leg of the given triangle is the mean proportional between the hypotenuse and the projection of that leg on the hypotenuse [3]

32 In the accompanying figure, $\triangle ABC$ is a scalene triangle. Angle $A = 51°$, side $AB = 17$ feet and side $AC = 26$ feet. BD is an altitude.



a Find the length of BD to the *nearest foot*. [3]

b Find the length of AD to the *nearest foot*. [3]

c Using the result obtained in part *b*, find the length of DC to the *nearest foot*. [1]

d Using results obtained in previous parts of this example, find angle C to the *nearest degree*. [3]

33 Triangle ABC is a right triangle with hypotenuse AB. CD is the median drawn from C. CE is the altitude drawn from C. Prove: $2(CD) \times CE = AC \times BC$ [*Suggestion:* In this proof, the fact may be assumed that the median to the hypotenuse of a right triangle is equal to one-half the hypotenuse.] [10]

34 The vertices of a parallelogram are A (5,2), B (9,4), C (11,8) and D (7,6).

a Using graph paper, draw $ABCD$. [1]

b Show that parallelogram $ABCD$ is a rhombus. [5]

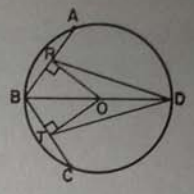
c Find the area of $ABCD$. [4]

35 In the accompanying figure, circle O has a diameter BD. Chord BA = chord BC. Lines OR and OT are drawn perpendicular to AB and BC, respectively. OR intersects AB at R and OT intersects BC at T. DR and DT are drawn.

Prove:

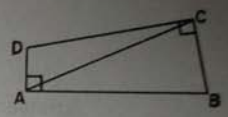
a $\angle ABD = \angle CBD$ [6]

b $\triangle RDB \cong \triangle TDB$ [4]



36 In the accompanying figure, $ABCD$ is a quadrilateral with $AB > BC$, $DA \perp AB$, $DC \perp BC$. Diagonal AC is drawn.

Prove: $DC > AD$ [10]



*37 Given: A (0,—4), B (4,4) and C (5,6).

a Find the slopes of line segments AB and AC. [2]

b Using your answers to part *a*, write a conclusion about the points A, B and C. [2]

c Write an equation of line AB. [3]

d Does the point (12,20) lie on the line AB? [1]

e What are the coordinates of the point where line AB crosses the x-axis? [2]

* This question is based on an optional topic in the syllabus.

FOR TEACHERS ONLY

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SCORING KEY
TENTH YEAR MATHEMATICS

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Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 22–27, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3 or 4.

(1) $9\sqrt{2}$
(2) (1,—4)
(3) 16
(4) $9\sqrt{3}$
(5) 2
(6) 22
(7) 54
(8) 30
(9) 9
(10) 12
(11) $\frac{5an}{2}$
(12) 12
(13) 81π
(14) 10
(15) 100
(16) 30
(17) 13
(18) $\frac{5}{2}$
(19) $x = 2$
(20) 6
(21) 38
(22) 1
(23) 4
(24) 4
(25) 2
(26) 3
(27) 2
(28) always
(29) never

Part II

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics*. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(32) *a* 13 [3]
b 11 [3]
c 15 [1]
d 41 [3]

(34) *c* 12 [4]

(37) *a* 2, 2 [2]
b Points *A*, *B* and *C* lie on the same straight line. [2]
c $y = 2x - 4$ [3]
d Yes [1]
e (2, 0) [2]