The University of the State of New York

Regents High School Examination

TENTH YEAR MATHEMATICS

Monday, August 21, 1961 — 8:30 to 11:30 a.m., only

Name of pupil..... Name of school.....

Name and author of textbook used.....

Name of teacher.....

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form.

1 The altitude of a trapezoid is 7 and the bases are 8 and 14. Find the area of the trapezoid. 1.....

2 The area of a circle is 16π. Find the length of the radius. 2.....

3 Two sides of an isosceles right triangle are each 2 units long. Find the length of the third side. 3.....

4 Two tangents from an external point to a circle intercept a minor arc of 160°. What is the number of degrees in the angle formed by the two tangents? 4.....

5 In circle O, central angle AOB equals 80°. If P is any point on major arc AB, find the number of degrees in angle APB. 5.....

6 In $\triangle ABC$, $AB = AC$ and the number of degrees in angle B is represented by x. Express in terms of x the number of degrees in an exterior angle at vertex A. 6.....

7 The coordinates of point A are $(-6, -4)$. If the y-axis is the perpendicular bisector of line segment AB, find the coordinates of point B. 7.....

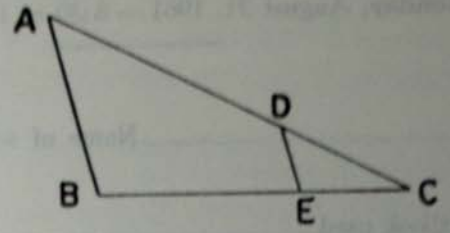
8 Find the length of the line segment joining the points whose coordinates are $(-4, 5)$ and $(0, 2)$. 8.....

9 In triangle ABC, side $AB = 18$, side $AC = 12$ and angle $A = 40°$. Find to the *nearest integer* the length of the altitude to side AB. 9.....

10 A diagonal of a certain rhombus is equal in length to a side of the rhombus. Find the number of degrees in one of the acute angles of the rhombus. 10.....

11 In right triangle ABC, CD is the altitude on the hypotenuse AB. If $AD = 5$ and $DB = 4$, find BC. 11.....

12 The accompanying diagram shows triangle ABC with D on AC and E on BC so that $DE \parallel AB$. If $BE = 8$, $EC = 4$ and $DE = 3$, find the length of AB.



12.....

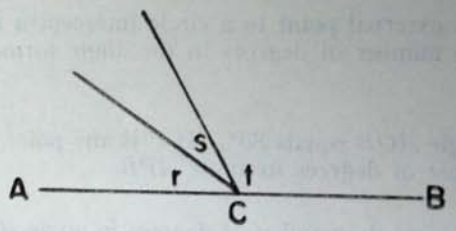
13 In circle O, diameter AB is perpendicular to chord CD at E. If diameter $AB = 13$ and $AE = 4$, find the length of CD. 13.....

14 In a circle whose circumference is 24π, find the length of an arc whose central angle is $30°$. 14.....

15 If the legs of a right triangle are 6 and 8, find the length of the radius of the circumscribed circle. 15.....

16 The ratio of the areas of two circles is 1:9. What is the ratio of the circumference of the smaller circle to the circumference of the larger circle? 16.....

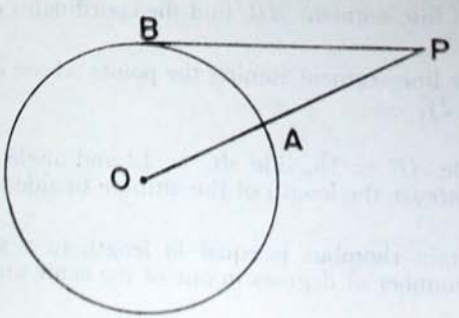
17 The accompanying diagram shows a straight line AB and angles r, s and t with common vertex C. If $r = 2s$ and $t = 6s$, find the number of degrees in angle s.



17.....

18 Side AB of triangle ABC is extended through B and forms an exterior angle of $65°$. Name the longest side of triangle ABC. 18.....

19 In the accompanying diagram, PB is tangent to the circle O at B. The line from O to P intersects the circle at A. If $OA = 5$ and $AP = 8$, find the length of BP.



19.....

Directions (20–25): Write on the line at the right of *each* of the following the *number* preceding the expression that best completes the statement or answers the question.

20 A mathematical system consists of (*a*) axioms and postulates, (*b*) theorems and (*c*) undefined terms. The correct order in which these occur in a logical system or structure such as geometry is

(1) *c*, *b*, *a*
(2) *a*, *b*, *c*
(3) *c*, *a*, *b*
(4) *a*, *c*, *b*

20.....

21 In a circle whose center is point O and whose radius is 4 inches, chord AB is drawn. How many points in the plane of circle O are equidistant from points A and B and 4 inches from point O?

(1) 1
(2) 2
(3) 3
(4) 4

21.....

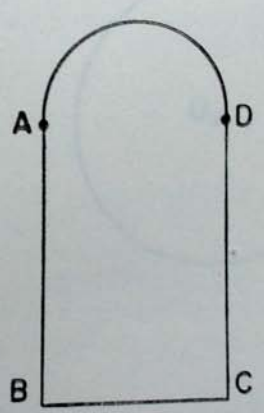
22 Given the statement: If a quadrilateral is a rectangle, then the quadrilateral is a parallelogram. Which statement is the inverse of the given statement?

(1) If a quadrilateral is a parallelogram, then the quadrilateral is a rectangle.
(2) If a quadrilateral is not a parallelogram, then the quadrilateral is not a rectangle.
(3) If a rectangle is a quadrilateral, then the rectangle is a parallelogram.
(4) If a quadrilateral is not a rectangle, then the quadrilateral is not a parallelogram.

22.....

23 In the accompanying figure, B and C are right angles and arc AD is a semicircle. If $AB = 12$ and $BC = 8$, then the perimeter of the figure is

(1) 40π
(2) 36π
(3) $32 + 4\pi$
(4) $32 + 8\pi$



23.....

24 The abscissa of point P (x, y) is equal to twice the ordinate of point P. An equation of the locus of point P is

(1) $x = 2y$
(2) $y = 2x$
(3) $x + y = 2$
(4) $x = 2$

24.....

25 Which of the following represents the order in which the statements below would be placed if they were arranged in the sequence in which they are postulated or proved?

a The area of a triangle is equal to one-half the product of a side and the altitude drawn to that side.

b The area of a rectangle is equal to the product of its base and altitude.

c The area of a regular polygon is equal to one-half the product of its perimeter and its apothem.

(1) *a*, *b*, *c* (3) *b*, *c*, *a*

(2) *a*, *c*, *b* (4) *b*, *a*, *c* 25.....

Directions (26–28): If the blank space in each statement below is replaced by the word *always*, *sometimes* (but not always) or *never*, the resulting statement will be true. Select the word that will correctly complete *each* statement and write this word on the line at the right.

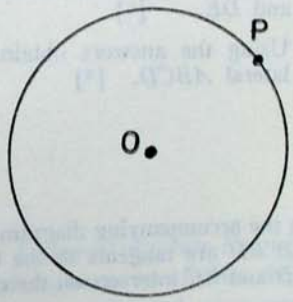
26 The exterior angle formed by extending the base of an isosceles triangle is ... an acute angle. 26.....

27 The median of a triangle divides the given triangle into two triangles which are ... equal in area. 27.....

28 If the diagonals of a quadrilateral are perpendicular, the quadrilateral is ... a rhombus. 28.....

Directions (29–30): Leave all construction lines on the paper.

29 Construct a tangent to circle O at point P.



30 Divide line segment AB into three equal parts.

Part II

Answer four questions from this part. Show all work unless otherwise directed.

31 Prove *either* *a* or *b*:

a A diameter perpendicular to a chord of a circle bisects the chord and its arcs. [10]

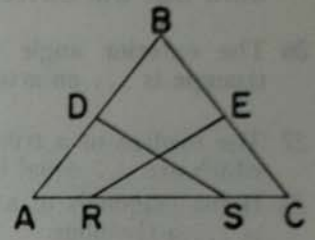
OR

b The area of a parallelogram is equal to the product of one side and the altitude drawn to that side. [10]

32 An equilateral triangle is inscribed in a circle. The perimeter of the triangle is 18. Find to the *nearest tenth* the area which is inside the circle but outside the triangle. [Use the approximations $\pi = 3.14$ and $\sqrt{3} = 1.73$.] [10]

33 The accompanying diagram shows triangle ABC with $AB = CB$, D the midpoint of AB, E the midpoint of CB and $AR = CS$. DS and ER are drawn.

Prove: $DS = ER$. [10]



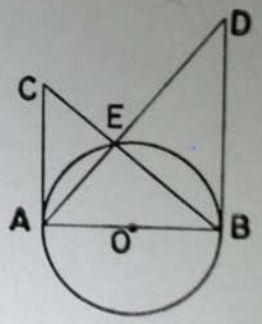
34 Given quadrilateral $ABCD$ with $AB \perp BC$ and $DC \perp BC$. $AD = 20$ inches, $DC = 40$ inches and angle $D = 36°$.

a Draw $AE \perp DC$ at E and find to the *nearest inch* the number of inches in the lengths of AE and DE. [6]

b Using the answers obtained in part *a*, find to the *nearest square inch* the area of quadrilateral $ABCD$. [4]

35 In the accompanying diagram, AB is a diameter of circle O and AC and BD are tangents to the circle at points A and B, respectively. AD and BC intersect on the circle at E.

Prove: $AC : AB = AB : BD$ [10]



36 Given the quadrilateral $ABCD$ whose vertices are A (4, 1), B (1, 3), C (—5, —6) and D (—2, —8).

a Using coordinate geometry, show that quadrilateral $ABCD$ is a rectangle. [8]

b Write a sentence indicating the special property (or properties) of a rectangle, which you used to show that quadrilateral $ABCD$ is a rectangle. [2]

FOR TEACHERS ONLY

10 INSTRUCTIONS FOR RATING TENTH YEAR MATHEMATICS

Monday, August 21, 1961 — 8:30 to 11:30 a.m., only

Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 20–25, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3 or 4.

(1) 77
(2) 4
(3) $2\sqrt{2}$ *or* 2.8
(4) 20
(5) 40
(6) $2x$
(7) $(6, -4)$
(8) 5
(9) 8
(10) 60
(11) 6
(12) 9
(13) 12
(14) 2π *or* 6.3
(15) 5
(16) 1 : 3
(17) 20
(18) AC
(19) 12
(20) 3
(21) 2
(22) 4
(23) 3
(24) 1
(25) 4
(26) never
(27) always
(28) sometimes

Part II

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics*. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(32) 22.1 [10]

(34) *a* $AE = 12, DE = 16$ [6]
b 384 [4]

(36) *b* A statement pertaining to the work done in part *a*, such as, "A parallelogram having one right angle is a rectangle." [2]