The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

TENTH YEAR MATHEMATICS

Tuesday, August 23, 1960 — 8:30 to 11:30 a.m., only

Name of pupil..... Name of school.....

Name and author of textbook used.....

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form.

1 Find the diagonal of a square the length of whose side is 10. 1.....

2 Two adjacent sides of a parallelogram are 8 and 10, and the included angle is 30°. Find the area of the parallelogram. 2.....

3 How many points are there 3 inches from a given line and also equidistant from two fixed points on the given line? 3.....

4 The radius of a circle is 5 inches. The area of a sector of the circle is 5π square inches. Find the number of degrees in the central angle of the sector. 4.....

5 In circle O, chords AB and CD intersect at E. If angle AED is 100° and arc AD is 150°, find the number of degrees in arc CB. 5.....

6 If the area of a trapezoid is 30 and the lengths of its bases are 10 and 2, find the length of its altitude. 6.....

7 Find the length of the line segment joining the points whose coordinates are $(-1, 3)$ and $(2, 5)$. 7.....

8 In triangles ABC and DEF, angle C equals angle E, $AC = EF$ and $BC = ED$. If $AB = 2x - 1$, $BC = 2x + 1$ and $FD = 5x - 4$, find the value of x. 8.....

9 The point $(6, 8)$ lies on a circle with center at the origin. Find the length of the radius of the circle. 9.....

10 Write an equation of the locus of points whose abscissas are -2. 10.....

11 In circle O, chord AC and central angle AOC are drawn. If the radius of the circle is 10 inches and central angle AOC is 50°, find to the *nearest inch* the distance from the center of the circle to chord AC. 11.....

12 The bases of an isosceles trapezoid are 15 and 9 and the nonparallel sides are each 5. Find the length of its altitude. 12.....

13 In triangle ABC, point D is on AB and point E is on BC so that DE is parallel to AC. If $AB = 12$, $BE = 8$ and $EC = 7$, find the length of DA. 13.....

[OVER]

14 The number of degrees in each interior angle of a regular polygon is twice the number of degrees in an exterior angle of the polygon. Find the number of sides of the polygon. 14.....

15 A regular polygon of 8 sides is equal in area to a triangle. If the apothem of the polygon equals the altitude of the triangle, and the base of the triangle is 24, find a side of the polygon. 15.....

16 A side of a rhombus is 17 and one diagonal is 16. Find the length of the other diagonal. 16.....

17 In a parallelogram, the number of degrees in the *unequal* angles is represented by $3x - 4$ and $2x + 9$. Find the number of degrees in the larger of the two angles. 17.....

18 Find the length of the side of an equilateral triangle whose area is $9\sqrt{3}$. 18.....

19 In circle O, diameter AB meets chord BC at B. If arc BC is 70°, find the number of degrees in angle ABC. 19.....

20 AB is tangent to circle O at point A, and secant BCD is drawn so that BD is 50 and BC is 2. Find the length of tangent AB. 20.....

21 A tree on level ground casts an 18-foot shadow at the same time that a 5-foot pole casts a 3-foot shadow. Find the number of feet in the height of the tree. 21.....

Directions (22–28): Write on the line at the right of *each* of the following the ***number*** preceding the expression that best completes the statement or answers the question.

22 A postulate is best defined as

(1) a statement which has been assumed (2) a statement which has been deduced (3) a statement which is obviously true (4) a statement which follows readily from a previously accepted statement 22.....

23 Given the following example of reasoning: "If a man is a good citizen, he pays his taxes. Mr. Smith pays his taxes. Therefore Mr. Smith is a good citizen."

Which statement describes the reasoning used in this example?

(1) The argument is not valid because it uses circular reasoning.
(2) The argument is not valid because it uses indirect reasoning.
(3) The argument is not valid because it uses reasoning from the converse.
(4) The argument is valid.

23.....

24 Which of the following represents the order in which the statements below would be placed if they were arranged in the sequence in which they were postulated or proved?

a An angle inscribed in a circle is measured by one-half its intercepted arc.
b A central angle is measured by its intercepted arc.
c An angle formed by two secants is measured by one-half the difference of the intercepted arcs.

(1) *a, b, c* (2) *a, c, b* (3) *b, a, c* (4) *c, b, a* 24.....

25 Which is a converse of the statement "If two parallel lines are cut by a transversal, corresponding angles are equal?"
(1) Corresponding angles of parallel lines are equal.
(2) If two lines are cut by a transversal and a pair of corresponding angles are equal, the lines are parallel.
(3) If two parallel lines are cut by a transversal, alternate interior angles are equal.
(4) Alternate interior angles of parallel lines are equal. 25.....

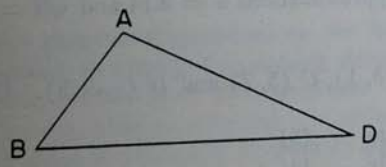
26 Which statement is true?
(1) A parallelogram inscribed in a circle must be a rectangle.
(2) A parallelogram inscribed in a circle must be a rhombus.
(3) A parallelogram inscribed in a circle must be a square.
(4) A parallelogram cannot be inscribed in a circle. 26.....

27 In a certain quadrilateral one pair of opposite sides are equal and parallel and one of the angles is a right angle. The quadrilateral *must* be a
(1) trapezoid (2) square (3) rhombus (4) rectangle 27.....

28 If the radius of a circle is doubled,
(1) the circumference and the area are each doubled
(2) the circumference is doubled and the area is multiplied by four
(3) the circumference and the area are each multiplied by four
(4) the area is doubled and the circumference is multiplied by four 28.....

Directions (29–30): Leave all construction lines on the paper.

29 Through A construct a line parallel to BD.



30 Construct the locus of the centers of circles which are tangent to line AB at P.

[3]

[OVER]

Part II

Answer four questions from this part. Show all work unless otherwise directed.

31 Prove either *a* or *b*:

a The sum of the angles of a triangle is equal to a straight angle. [10]

OR

b An angle formed by two chords intersecting inside the circle is measured by one-half the sum of the intercepted arcs. [10]

32 In circle O, point E lies between A and O on diameter AB. Chord CD is perpendicular to AB at E. CA, CB and DB are drawn.

Prove: $EB \times AB = CB \times DB$ [10]

33 Triangle ABC is isosceles, with vertex angle C. CA is extended through A to D, and CB is extended through B to E, so that $AD > BE$. Line DE is drawn.

a Prove: angle $CED >$ angle CDE [6]

b If angle CDE contains x degrees and angle CED is 10 degrees more than angle CDE, express angle CAB in terms of x. [4]

34 An observer stands at a window so that his eye height is 13 feet above a level street. He notes that the angle of depression of the foot of a building across the street is 20° and that the angle of elevation of the top of the same building is 35°. Find to the *nearest foot* the height of the building. [10]

35 A circle is inscribed in an equilateral triangle whose side is 12 inches. Another triangle is formed within the circle by joining the points of tangency of the circle and the original triangle. Find, to the *nearest square inch*, the area of that part of the circle which is *not* included in the inner triangle. [Use the approximations $\pi = 3.14$ and $\sqrt{3} = 1.73$.] [10]

36 Given: A (—5, —1), B (3, 1), C (5, 7) and D (—3, 5). Using coordinate geometry, show that

a $ABCD$ is a parallelogram [6]

b $ABCD$ is not a rectangle [4]

*37 *a* Plot points A (2, 1), B (6, 7), C (4, 9). [1]

b Find the coordinates of the midpoint D of AC and the midpoint E of BC. [2]

c Draw DE. Find the slope of DE. [2]

d Find the slope of AB. [2]

e Draw a conclusion from your answers to parts *c* and *d*, and quote a theorem of which this is a particular example. [1, 2]

*This question is based on one of the optional topics in the syllabus.

FOR TEACHERS ONLY

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INSTRUCTIONS FOR RATING TENTH YEAR MATHEMATICS

Tuesday, August 23, 1960 — 8:30 to 11:30 a.m., only

Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions *22–28*, allow credit if the pupil has written the correct answer instead of the number *1*, *2*, *3* or *4*.

(1) $10\sqrt{2}$ *or* 14.1
(2) 40
(3) two
(4) 72
(5) 50
(6) 5
(7) $\sqrt{13}$ *or* 3.6
(8) 1
(9) 10
(10) $x = -2$
(11) 9
(12) 4
(13) $5\frac{3}{8}$
(14) 6
(15) 3
(16) 30
(17) 101
(18) 6
(19) 55
(20) 10
(21) 30
(22) 1
(23) 3
(24) 3
(25) 2
(26) 1
(27) 4
(28) 2

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics*. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

Part II

(33) *b* $x + 5$ [4]

(34) 38 [10]

(35) 22 [10]

*(37) *b* (3, 5) and (5, 8) [2]

c $\frac{3}{2}$ [2]

d $\frac{3}{2}$ [2]

e $DE \parallel AB$ [1]
and "A line joining the midpoints of two sides of a triangle is parallel to the third side." [2]