The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

TENTH YEAR MATHEMATICS

Monday, June 18, 1984 — 1:15 to 4:15 p.m., only

The last page of the booklet is the answer sheet. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

On page 9 you will find the "Tables of Natural Trigonometric Functions" which you may need to answer some questions in this examination. Fold this page along the perforations, and tear it off also slowly and carefully.

When you have completed the examination, you must sign the statement printed at the end of the answer paper, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer paper cannot be accepted if you fail to sign this declaration.

**DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN**

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form. Write your answers in the spaces provided on the separate answer sheet.

1 Secants $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ intersect circle O at A, B, C, and D. If $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$, $m\widehat{AB} = 100$, and $m\widehat{AC} = 100$, what is $m\widehat{CD}$?
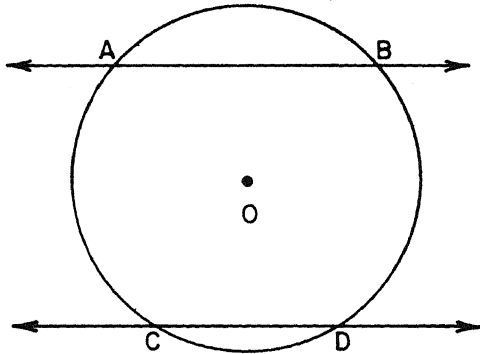

2 In rectangle $ABCD$, the coordinates of three of the vertices are $A(1,2)$, $B(5,2)$, and $C(5,5)$. Find the coordinates of point D.

3 In the accompanying figure, regular hexagon $ABCDEF$ is inscribed in circle O. Diagonals $\overline{AC}$ and $\overline{BOE}$ of the hexagon intersect at point G. What is $m\angle EGC$?
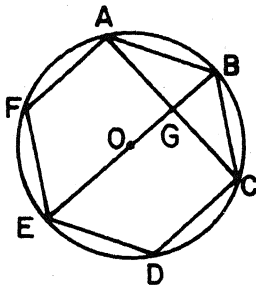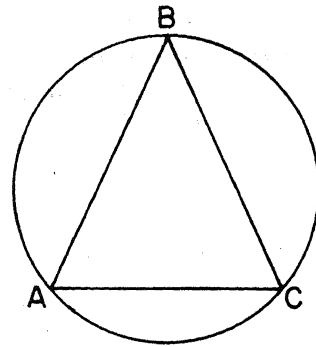

4 In a triangle, the ratio of the measures of the angles is $4{:}5{:}6$. Find the number of degrees in the measure of the *smallest* angle of the triangle.

5 In the accompanying diagram, $\overleftrightarrow{AB}$, $\overleftrightarrow{CD}$, and $\overleftrightarrow{FH}$ intersect at E, and $\overleftrightarrow{AB}$ bisects $\angle FEC$. If $m\angle DEB = 25$, what is $m\angle DEF$?
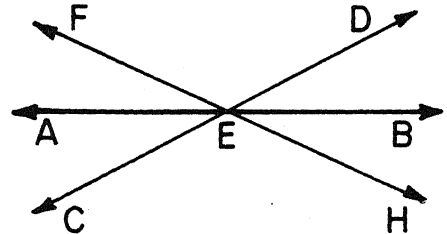
6 In the accompanying diagram, $\triangle ABC$ is inscribed in the circle. If $\angle A \cong \angle C$ and $m\widehat{AC} = 100$, find $m\angle C$.

7 Triangle DEF is inscribed in circle O. If $m\angle D = 85$ and $m\angle E = 65$, which side of triangle DEF lies nearest to the center of circle O?

8 In a circle, chords $\overline{AB}$ and $\overline{CD}$ intersect at point E. If $AE = 3$, $BE = 8$, and $CE = 2$, find DE.

9 A vertical antenna 75 meters tall casts a shadow 35 meters long. At the same time, a flagpole nearby casts a shadow 14 meters long. What is the number of meters in the height of the flagpole?

10 In the accompanying diagram of $\triangle ABC$, $\overline{DE} \parallel \overline{AB}$, $CD = 3$, $AD = 4$, and $EB = 8$. Find CB.
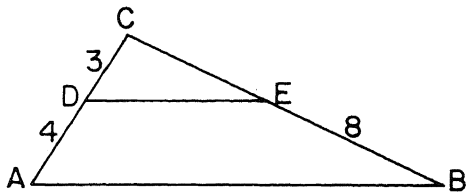

11 The coordinates of the vertices of $\triangle ABC$ are $A(1,3)$, $B(-2,-1)$, and $C(4,-3)$. Find the coordinates of the point where the median from A intersects side $\overline{BC}$.

12 In a circle of radius 5, a chord of length 8 is drawn. Find the distance from the center of the circle to the chord.

Directions (13–29): Write in the space provided on the separate answer sheet the *numeral* preceding the expression that best completes *each* statement or answers *each* question.

13 In $\triangle ABC$, $m\angle A = 40$ and $m\angle B = 60$. What is the measure of an exterior angle at vertex C?
(1) 40°   (3) 80°
(2) 60°   (4) 100°

14 In the accompanying diagram, triangle ABC is similar to triangle XYZ, $AB:XY = 5:3$, and $BC = 20$. Which statement *must* be true?
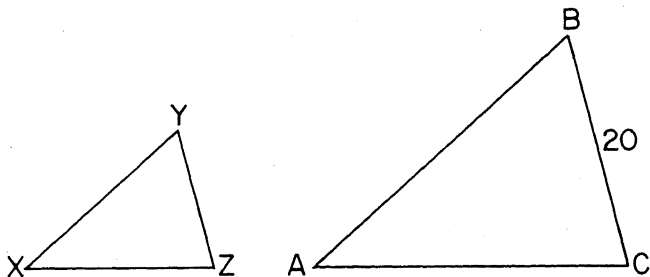
(1) $YZ = 12$ (3) $XY = 3$
(2) $XZ = 12$ (4) $AC < 20$

15 What is the perimeter of a rhombus whose diagonals are 12 and 16?
(1) 10   (3) 40
(2) 20   (4) 100

16 What is the slope of any line parallel to the line whose equation is $y = -\frac{2}{3}x + 4$?
(1) $-\frac{2}{3}$ (3) $-\frac{3}{2}$
(2) $\frac{2}{3}$ (4) $\frac{3}{2}$

17 One circle is inscribed in an equilateral triangle and another circle is circumscribed about the same equilateral triangle. What is the ratio of the radius of the inscribed circle to the radius of the circumscribed circle?
(1) $1:\sqrt{3}$ (3) $1:2$
(2) $1:\sqrt{2}$ (4) $2:\sqrt{3}$

18 In $\triangle ABC$, $AB = 8$ and $BC = 6$. Which statement is *always* true?
(1) $AC > 14$
(2) $AC < 2$
(3) $AC < 2$ and $AC > 14$
(4) $AC > 2$ and $AC < 14$

19 Each of two circles has a radius of 5 centimeters. Their centers are 8 centimeters apart. The total number of common tangents that can be drawn to these circles is
(1) 1   (3) 3
(2) 2   (4) 4

20 What is the converse of the statement, "If John buys the shirt, then the shirt is cotton"?
(1) If John does not buy the shirt, then the shirt is not cotton.
(2) If the shirt is cotton, then John buys the shirt.
(3) If the shirt is cotton, then John does not buy the shirt.
(4) If the shirt is not cotton, then John does not buy the shirt.

21 As shown in the accompanying diagram, a circle with a diameter of length 6 is inscribed in a square. What is the area of the shaded portion of the figure?
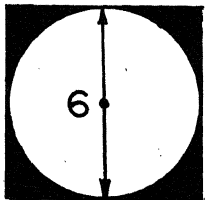
(1) 36
(2) 9π
(3) $36 + 9\pi$
(4) $36 - 9\pi$

22 Two parallel lines are cut by a transversal such that two interior angles on the same side of the transversal measure $4x$ and $5x$ degrees. The measure of the *smaller* angle is
(1) 10°
(2) 20°
(3) 80°
(4) 40°

23 Which property is *not* true for all rhombuses?
(1) All sides are congruent.
(2) The diagonals are perpendicular to each other.
(3) All angles are congruent.
(4) The diagonals divide the figure into four congruent triangles.

24 In right triangle ABC, $m\angle C = 90$, $BC = 3$, and $AC = 4$. What is sin B?
(1) $\frac{3}{4}$
(2) $\frac{4}{5}$
(3) $\frac{3}{5}$
(4) $\frac{4}{3}$

25 The corresponding altitudes of two similar triangles measure 4 and 9. The ratio of the perimeters of the two triangles is
(1) $\sqrt{2}:\sqrt{3}$
(2) 2:3
(3) 16:81
(4) 4:9

26 How many points are equidistant from 2 parallel lines and also equidistant from 2 points on one of these lines?
(1) 1
(2) 2
(3) 3
(4) 4

27 In the accompanying diagram, $\overline{PA}$ and $\overline{PB}$ are tangent to circle O and $m\angle AOP = 35$. What is $m\angle APB$?
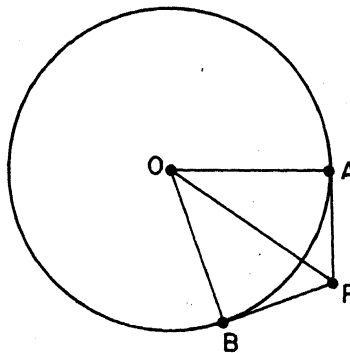
(1) 70
(2) 55
(3) 110
(4) 145

28 In isosceles trapezoid $ABCD$ with bases $\overline{AB}$ and $\overline{CD}$, $m\angle D = 45$. If $AB = 12$ and $DC = 20$, then the length of an altitude of trapezoid $ABCD$ is
(1) 12
(2) $2\sqrt{2}$
(3) 8
(4) 4

29 Two right triangles are *not* necessarily congruent if
(1) the hypotenuse and a leg of one triangle are congruent to the corresponding parts of the other triangle
(2) the hypotenuse and an acute angle of one triangle are congruent to the corresponding parts of the other triangle
(3) the corresponding acute angles of the triangles are congruent
(4) two legs of one triangle are congruent to two legs of the other triangle

Directions (30): Leave all construction lines on the answer sheet.

30 *On the answer sheet,* construct the altitude of trapezoid $ABCD$ from B. Label the altitude $\overline{BE}$.

Answers to the following questions are to be written on paper provided by the school.

Part II

Answer four questions from this part. Show all work unless otherwise directed.
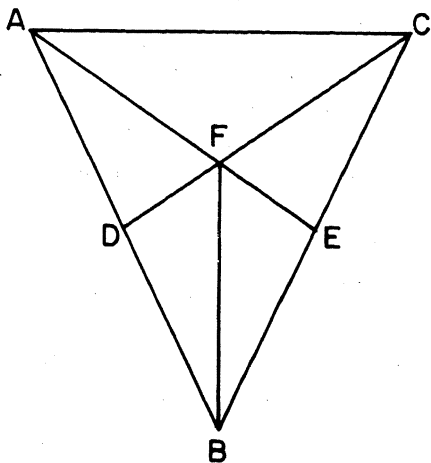
31 Prove *either* a or b but *not* both.
  a If the three angles of one triangle are congruent to the three angles of another triangle, the triangles are similar.   [10]

OR

  b The square of the length of the hypotenuse of a right triangle is equal to the sum of the squares of the lengths of the legs.   [10]

32 Given: $\triangle ABC$, $\overline{ADB}$, $\overline{CEB}$, $\overline{AE}$ and $\overline{DC}$ intersect at F, $\overline{DF} \cong \overline{EF}$, and $\angle EFB \cong \angle DFB$.

Prove: $\triangle ABC$ is isosceles.   [10]

33 Given: $\overleftrightarrow{RS}$ and point T located 10 units from $\overleftrightarrow{RS}$.
  a Describe fully the locus of points at a distance of 4 units from $\overleftrightarrow{RS}$.   [3]
  b Describe fully the locus of points at a distance d units from point T.   [3]
  c How many points will satisfy the conditions of both parts a and b if:
    (1) $d = 5$ [2]
    (2) $d = 7$ [2]

34 Given: $\overleftrightarrow{GAF}$ is tangent to circle O at A, $\overline{AD}$ is a diameter, $\overline{GBC}$, $\overline{ARD}$, $\overline{CRE}$, $m\widehat{AB} = 60$, $m\widehat{CD} = 40$, and $m\widehat{BC} = m\widehat{AE}$.
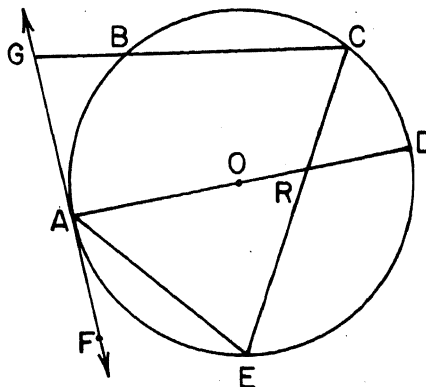
Find: a $m\widehat{ED}$ [2]
      b $m\angle AEC$ [2]
      c $m\angle FAE$ [2]
      d $m\angle ARC$ [2]
      e $m\angle CGA$ [2]

35 Given: $\triangle ABD$, C is a point on $\overline{BD}$ such that $AC > AB$.
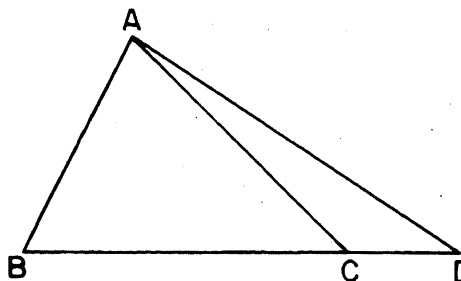
Prove: $AD > AB$ [10]

☞ GO RIGHT ON TO THE NEXT PAGE.

36 A regular pentagon, each of whose sides has length 20, is inscribed in a circle.
   a Find, to the *nearest whole number*, the apothem of the pentagon. [6]
   b Using your answer from part *a*, find the area of the pentagon. [4]

*37 Using coordinate geometry, show that quadrilateral $QRST$ with coordinates $Q(-5,2)$, $R(7,6)$, $S(8,3)$, and $T(-4,-1)$, is a rectangle and state a reason for your conclusion. [10]

*This question is based on an optional topic in the syllabus.

Tables of Natural Trigonometric Functions
(For use with 9th and 10th Year Mathematics Regents Examinations)

Angle	Sine	Cosine	Tangent	Angle	Sine	Cosine	Tangent
1°	.0175	.9998	.0175	46°	.7193	.6947	1.0355
2°	.0349	.9994	.0349	47°	.7314	.6820	1.0724
3°	.0523	.9986	.0524	48°	.7431	.6691	1.1106
4°	.0698	.9976	.0699	49°	.7547	.6561	1.1504
5°	.0872	.9962	.0875	50°	.7660	.6428	1.1918
6°	.1045	.9945	.1051	51°	.7771	.6293	1.2349
7°	.1219	.9925	.1228	52°	.7880	.6157	1.2799
8°	.1392	.9903	.1405	53°	.7986	.6018	1.3270
9°	.1564	.9877	.1584	54°	.8090	.5878	1.3764
10°	.1736	.9848	.1763	55°	.8192	.5736	1.4281
11°	.1908	.9816	.1944	56°	.8290	.5592	1.4826
12°	.2079	.9781	.2126	57°	.8387	.5446	1.5399
13°	.2250	.9744	.2309	58°	.8480	.5299	1.6003
14°	.2419	.9703	.2493	59°	.8572	.5150	1.6643
15°	.2588	.9659	.2679	60°	.8660	.5000	1.7321
16°	.2756	.9613	.2867	61°	.8746	.4848	1.8040
17°	.2924	.9563	.3057	62°	.8829	.4695	1.8807
18°	.3090	.9511	.3249	63°	.8910	.4540	1.9626
19°	.3256	.9455	.3443	64°	.8988	.4384	2.0503
20°	.3420	.9397	.3640	65°	.9063	.4226	2.1445
21°	.3584	.9336	.3839	66°	.9135	.4067	2.2460
22°	.3746	.9272	.4040	67°	.9205	.3907	2.3559
23°	.3907	.9205	.4245	68°	.9272	.3746	2.4751
24°	.4067	.9135	.4452	69°	.9336	.3584	2.6051
25°	.4226	.9063	.4663	70°	.9397	.3420	2.7475
26°	.4384	.8988	.4877	71°	.9455	.3256	2.9042
27°	.4540	.8910	.5095	72°	.9511	.3090	3.0777
28°	.4695	.8829	.5317	73°	.9563	.2924	3.2709
29°	.4848	.8746	.5543	74°	.9613	.2756	3.4874
30°	.5000	.8660	.5774	75°	.9659	.2588	3.7321
31°	.5150	.8572	.6009	76°	.9703	.2419	4.0108
32°	.5299	.8480	.6249	77°	.9744	.2250	4.3315
33°	.5446	.8387	.6494	78°	.9781	.2079	4.7046
34°	.5592	.8290	.6745	79°	.9816	.1908	5.1446
35°	.5736	.8192	.7002	80°	.9848	.1736	5.6713
36°	.5878	.8090	.7265	81°	.9877	.1564	6.3138
37°	.6018	.7986	.7536	82°	.9903	.1392	7.1154
38°	.6157	.7880	.7813	83°	.9925	.1219	8.1443
39°	.6293	.7771	.8098	84°	.9945	.1045	9.5144
40°	.6428	.7660	.8391	85°	.9962	.0872	11.4301
41°	.6561	.7547	.8693	86°	.9976	.0698	14.3007
42°	.6691	.7431	.9004	87°	.9986	.0523	19.0811
43°	.6820	.7314	.9325	88°	.9994	.0349	28.6363
44°	.6947	.7193	.9657	89°	.9998	.0175	57.2900
45°	.7071	.7071	1.0000	90°	1.0000	.0000	

| Part I Score......... .... |
| Part II Score........ .... |
| Total .... |
| Rater's Initials: .... |

ANSWER SHEET

Pupil ....Teacher ....

School ....

Name and author of textbook used ....

Your answers to Part I should be recorded on this answer sheet.

Part I
Answer all questions in this part.

1......................... 11......................... 21.........................

2......................... 12......................... 22.........................

3......................... 13......................... 23.........................

4......................... 14......................... 24.........................

5......................... 15......................... 25.........................

6......................... 16......................... 26.........................

7......................... 17......................... 27.........................

8......................... 18......................... 28.........................

9......................... 19......................... 29.........................

10......................... 20......................... 30 **Answer question 30 on the other side of this sheet.**
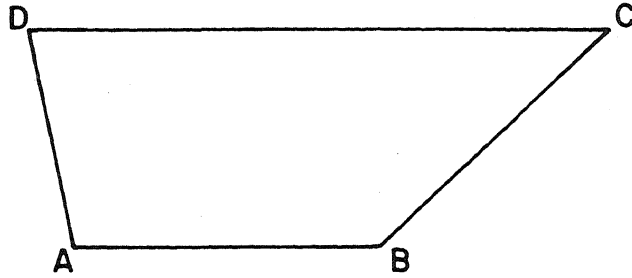
Your answers for Part II should be placed on paper provided by the school.

The declaration below should be signed when you have completed the examination.

I do hereby affirm, at the close of this examination, that I had no unlawful knowledge of the questions or answers prior to the examination, and that I have neither given nor received assistance in answering any of the questions during the examination.

_____

Signature

Math. 10–June '84   [12]

FOR TEACHERS ONLY

Use only *red* ink or *red* pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 13–29, allow credit if the pupil has written the correct answer instead of the numeral 1, 2, 3, or 4.

(1) 60

(2) (1,5) or $\begin{array}{l} x = 1 \\ y = 5 \end{array}$

(3) 90

(4) 48

(5) 130

(6) 65

(7) $\overline{EF}$ or EF or d

(8) 12

(9) 30

(10) 14

(11) (1,−2) or $\begin{array}{l} x = 1 \\ y = -2 \end{array}$

(12) 3

(13) 4

(14) 1

(15) 3

(16) 1

(17) 3

(18) 4

(19) 2

(20) 2

(21) 4

(22) 3

(23) 3

(24) 2

(25) 4

(26) 1

(27) 3

(28) 4

(29) 3

(30) construction

[OVER]

Part II

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics*. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(33) *a* two lines parallel to $\overleftrightarrow{RS}$, 4 units from $\overleftrightarrow{RS}$, and on either side of $\overleftrightarrow{RS}$ [3]

     b a circle with center at T having a radius of d units   [3]

     c (1) 0   [2]
         (2) 2   [2]

(34) *a* 100   [2]
     b 70   [2]
     c 40   [2]
     d 120   [2]
     e 80   [2]

(36) *a* 14   [6]
     b 700   [4]