The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

TENTH YEAR MATHEMATICS

Friday, June 18. 1976 — 1:15 to 4:15 p.m., only

The last page of the booklet is the answer sheet. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

On page 5 you will find the "Tables of Natural Trigonometric Functions" which you may need to answer some questions in this examination. Fold this page along the perforations, and tear it off also slowly and carefully.

When you have completed the examination, you must sign the statement printed at the end of the answer paper, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer paper cannot be accepted if you fail to sign this declaration.

**DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN**

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form. Write your answers in the spaces provided on the separate answer sheet.

1 In two similar triangles, the ratio of the lengths of a pair of corresponding sides is 5:8. If the perimeter of the larger triangle is 32, find the perimeter of the *smaller* triangle.

2 In the accompanying figure, O is the center of the circle, and $\overline{AEOB}$ and $\overline{ACD}$ are secants. Radii $\overline{OC}$ and $\overline{OD}$ are drawn. If $\overline{AC} \cong \overline{CO}$ and $m\angle A = 30$, find $m\angle BOD$.
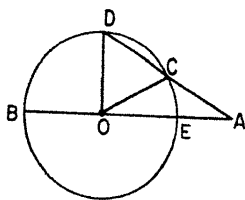
3 In the accompanying diagram, $\overline{BDC}$ and $\overline{DE} \perp \overline{AEC}$. If $m\angle BDA = 80$ and $m\angle EDC = 50$, find $m\angle DAC$.
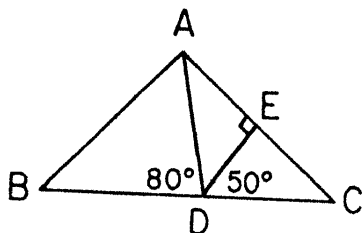
4 In triangle RST, $m\angle R = 62$ and the measure of an exterior angle at T is 119°. Which is the *longest* side of the triangle?

5 In parallelogram $ABCD$, $m\angle A = \dfrac{1}{2}m\angle B$. Find $m\angle A$.

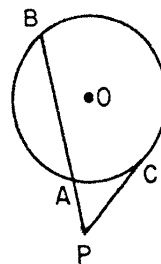6 The circumference of a circle is 12π. Find the length of an arc whose measure is 90°.

7 Find the sum of the measures of the interior angles of a polygon with seven sides.

8 The diagonals of parallelogram $ABCD$ intersect at point E. If the coordinates of the vertices of the parallelogram are $A(0,1)$, $B(3,1)$, $C(4,5)$, and $D(1,5)$, find the coordinates of E.

9 In the accompanying figure, points D and E are the midpoints of sides $\overline{AB}$ and $\overline{AC}$, respectively, of $\triangle ABC$. If $m\angle A = 30$ and $m\angle C = 45$, find $m\angle ADE$.
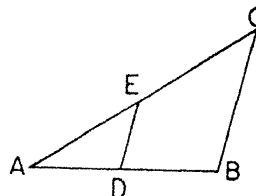
10 In the accompanying diagram, $\overline{PAB}$ is a secant and $\overline{PC}$ is a tangent to circle O. If $PB = 16$ and $PA = 4$, find PC.


11 Express in terms of π the circumference of a circle whose *diameter* is 3.

12 The lengths of the sides of $\triangle PQR$ are 7, 10, and 15, respectively. Find the perimeter of the triangle formed by joining the midpoints of the sides of $\triangle PQR$.

13 What are the coordinates of the point on the y-axis equidistant from points $(2,4)$ and $(2,-4)$?

14 The area of square $ABCD$ as shown in the accompanying diagram is 121. $\overline{EKF}$ is drawn parallel to $\overline{AD}$ and $\overline{GKH}$ is drawn parallel to $\overline{AB}$ so as to form the smaller squares $AGKE$ and $FCHK$. If the area of $AGKE$ is 36, find the area of $FCHK$.
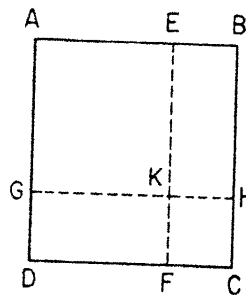
15 Find the area of a trapezoid whose bases have lengths 8 and 12, respectively, and whose altitude is 6.

16 In the accompanying figure of circle O, chord $\overline{AB}$ is parallel to diameter $\overline{COD}$. If $AB = 6$ and $CD = 10$, how far apart are the two line segments?
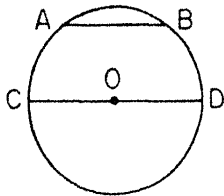
17 The measure of a base angle of an isosceles triangle is 30°. If the length of a leg of the triangle is 12, what is the length of the altitude drawn from the vertex angle to the base?

18 Find the length of a diagonal of a square whose area is 8.

19 The lengths of the sides of a right triangle are 7, 24, and 25, respectively. Express as a fraction the sine of the *smallest* angle of the triangle.

Directions (20–29): Write in the space provided on the separate answer sheet the *numeral* preceding the expression that best completes *each* statement or answers *each* question.

20 A parallelogram must be a square if the diagonals are
(1) congruent and bisect the angles to which they are drawn
(2) congruent and do not bisect the angles to which they are drawn
(3) not congruent and bisect the angles to which they are drawn
(4) not congruent and do not bisect the angles to which they are drawn

21 What is the total number of points in the plane of a rectangle that are equidistant from the four vertices of the rectangle?
(1) 1   (3) 3
(2) 2   (4) 4

22 Which is an equation of the locus of points whose ordinates exceed twice their abscissas by 3?
(1) $x = 2y - 3$ (3) $y = 2x - 3$
(2) $x = 2y + 3$ (4) $y = 2x + 3$

23 Consider the following argument: Given two real numbers, x and y. If x is *not* less than y and also x is *not* greater than y, then x equals y. This argument is an example of
(1) circular reasoning
(2) indirect reasoning
(3) reasoning from the inverse
(4) reasoning from the converse

24 Which set of integers can *not* represent the respective lengths of the sides of a triangle?
(1) $\{10, 11, 12\}$ (3) $\{3, 7, 10\}$
(2) $\{8, 14, 18\}$ (4) $\{7, 7, 13\}$

25 If in trapezoid $ABCD$, $\overline{AB} \parallel \overline{DC}$ and diagonals $\overline{AC}$ and $\overline{BD}$ intersect at point E, then $\triangle ABE$ and $\triangle CDE$ must always be
(1) congruent triangles   (3) right triangles
(2) scalene triangles   (4) similar triangles

26 Regular hexagon $ABCDEF$ is inscribed in circle O and chords $\overline{AC}$ and $\overline{AD}$ are drawn. Which is a valid conclusion?
(1) $\overline{AC}$ is perpendicular to $\overline{CD}$.
(2) $\overline{AC}$ is perpendicular to $\overline{AD}$.
(3) $\overline{AC}$ is congruent to $\overline{CD}$.
(4) $\overline{AC}$ is congruent to $\overline{AD}$.

27 The vertices of $\triangle ABC$ are $A(0,0)$, $B(-4,3)$, and $C(6,0)$. What is the slope of $\overleftrightarrow{AB}$?
(1) $\dfrac{3}{4}$ (3) $-\dfrac{3}{4}$

(2) $\dfrac{4}{3}$ (4) $-\dfrac{4}{3}$

28 In the accompanying figure, $\overline{AB}$ is tangent to circle O at E, $\overline{DE}$, $\overline{EF}$, and $\overline{FD}$ are chords and $m\angle AEF = 136$. What is $m\angle D$?
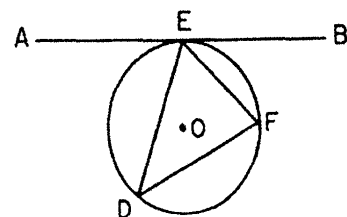
(1) 22   (3) 68
(2) 44   (4) 88

29 In $\triangle PRT$, K is a point on $\overline{TP}$ and G is a point on $\overline{TR}$ such that $\overline{KG} \parallel \overline{PR}$. If $TP = 20$, $KP = 4$, and $GR = 7$, what is TG?

(1) 16   (3) 28

(2) 17   (4) 35

Directions (30) Leave all construction lines on the answer sheet.

30 By construction *on the answer sheet*, find the center of the circle which can be circumscribed about $\triangle ABC$.

Answers to the following questions are to be written on paper provided by the school.

Part II

Answer four questions from this part. Show all work unless otherwise directed.
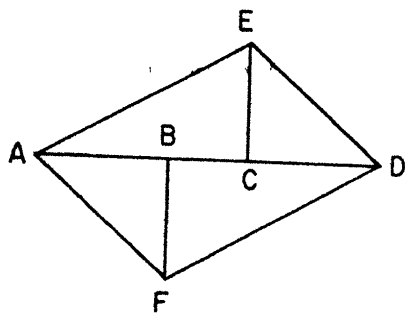
31 Prove *either a* or *b*, but *not* both:

a The area of a trapezoid is equal to one-half the product of the length of the altitude and the sum of the lengths of the bases. [10]
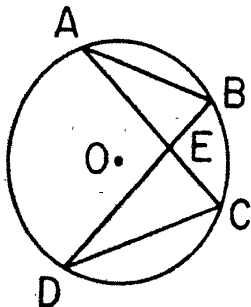
OR

b The square of the length of the hypotenuse of a right triangle is equal to the sum of the squares of the lengths of the legs. [10]

32 Given: $\overline{ABCD}$, $\overline{EC} \perp \overline{AD}$, $\overline{FB} \perp \overline{AD}$, $\overline{BF} \cong \overline{CE}$, and $\overline{CD} \cong \overline{BA}$

Prove: $\overline{AE} \parallel \overline{FD}$ [10]

33 Given: circle O, chords $\overline{AB}$, $\overline{CD}$, $\overline{AEC}$, $\overline{BED}$
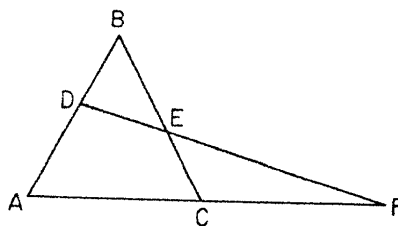

a Prove: $\triangle BAE \sim \triangle CDE$ [6]

b If $BE = 3$ and $CE = 4$, find the numerical value of

(1) $\dfrac{BA}{CD}$ [2]

(2) $\dfrac{\text{Area } \triangle AEB}{\text{Area } \triangle DEC}$ [2]

34 Given: $\triangle ABC$, $\overline{ACF}$, $\overline{DEF}$, $\overline{AB} \cong \overline{BC}$

Prove: $DF > AD$ [10]

35 *a* Find the area of $\triangle ABC$ whose vertices are $A(1,7)$, $B(7,6)$, and $C(3,11)$. [7]

b Find, in radical form, the length of the median from B to $\overline{AC}$. [3]

36 Given: Circle O with secants $\overline{PAF}$ and $\overline{PDC}$. $\overline{PF} \parallel \overline{BD}$, chords $\overline{AC}$ and $\overline{BD}$ intersect at E.
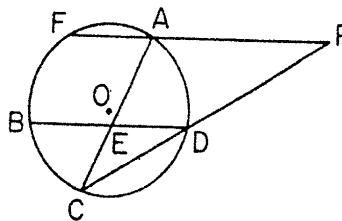
If $m\overparen{BF} = 70$, $m\overparen{FA} = 60$, and $m\overparen{BC}:m\overparen{CD} = 3:5$, find

(1) $m\overparen{AD}$ [2]   (4) $m\angle P$ [2]

(2) $m\overparen{BC}$ [2]   (5) $m\angle BDP$ [2]

(3) $m\angle AED$ [2]

*37 The vertices of parallelogram $STWU$ are $S(1,1)$, $T(-2,3)$, $W(0,b)$, and $U(3,-5)$.

a Find the slope of $\overline{ST}$.   [2]

b Express the slope of $\overline{UW}$ in terms of b.   [3]

c Find the value of b.   [2]

d Write an equation of the line passing through point S and perpendicular to $\overline{ST}$.   [3]

* This question is based on an optional topic in the syllabus.

Tables of Natural Trigonometric Functions
(For use with 9th and 10th Year Mathematics Regents Examinations)

Angle	Sine	Cosine	Tangent	Angle	Sine	Cosine	Tangent
1°	.0175	.9998	.0175	46°	.7193	.6947	1.0355
2°	.0349	.9994	.0349	47°	.7314	.6820	1.0724
3°	.0523	.9986	.0524	48°	.7431	.6691	1.1106
4°	.0698	.9976	.0699	49°	.7547	.6561	1.1504
5°	.0872	.9962	.0875	50°	.7660	.6428	1.1918
6°	.1045	.9945	.1051	51°	.7771	.6293	1.2349
7°	.1219	.9925	.1228	52°	.7880	.6157	1.2799
8°	.1392	.9903	.1405	53°	.7986	.6018	1.3270
9°	.1564	.9877	.1584	54°	.8090	.5878	1.3764
10°	.1736	.9848	.1763	55°	.8192	.5736	1.4281
11°	.1908	.9816	.1944	56°	.8290	.5592	1.4826
12°	.2079	.9781	.2126	57°	.8387	.5446	1.5399
13°	.2250	.9744	.2309	58°	.8480	.5299	1.6003
14°	.2419	.9703	.2493	59°	.8572	.5150	1.6643
15°	.2588	.9659	.2679	60°	.8660	.5000	1.7321
16°	.2756	.9613	.2867	61°	.8746	.4848	1.8040
17°	.2924	.9563	.3057	62°	.8829	.4695	1.8807
18°	.3090	.9511	.3249	63°	.8910	.4540	1.9626
19°	.3256	.9455	.3443	64°	.8988	.4384	2.0503
20°	.3420	.9397	.3640	65°	.9063	.4226	2.1445
21°	.3584	.9336	.3839	66°	.9135	.4067	2.2460
22°	.3746	.9272	.4040	67°	.9205	.3907	2.3559
23°	.3907	.9205	.4245	68°	.9272	.3746	2.4751
24°	.4067	.9135	.4452	69°	.9336	.3584	2.6051
25°	.4226	.9063	.4663	70°	.9397	.3420	2.7475
26°	.4384	.8988	.4877	71°	.9455	.3256	2.9042
27°	.4540	.8910	.5095	72°	.9511	.3090	3.0777
28°	.4695	.8829	.5317	73°	.9563	.2924	3.2709
29°	.4848	.8746	.5543	74°	.9613	.2756	3.4874
30°	.5000	.8660	.5774	75°	.9659	.2588	3.7321
31°	.5150	.8572	.6009	76°	.9703	.2419	4.0108
32°	.5299	.8480	.6249	77°	.9744	.2250	4.3315
33°	.5446	.8387	.6494	78°	.9781	.2079	4.7046
34°	.5592	.8290	.6745	79°	.9816	.1908	5.1446
35°	.5736	.8192	.7002	80°	.9848	.1736	5.6713
36°	.5878	.8090	.7265	81°	.9877	.1564	6.3138
37°	.6018	.7986	.7536	82°	.9903	.1392	7.1154
38°	.6157	.7880	.7813	83°	.9925	.1219	8.1443
39°	.6293	.7771	.8098	84°	.9945	.1045	9.5144
40°	.6428	.7660	.8391	85°	.9962	.0872	11.4301
41°	.6561	.7547	.8693	86°	.9976	.0698	14.3007
42°	.6691	.7431	.9004	87°	.9986	.0523	19.0811
43°	.6820	.7314	.9325	88°	.9994	.0349	28.6363
44°	.6947	.7193	.9657	89°	.9998	.0175	57.2900
45°	.7071	.7071	1.0000	90°	1.0000	.0000	

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The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

TENTH YEAR MATHEMATICS

Friday, June 18, 1976 — 1:15 to 4:15 p.m., only

ANSWER SHEET

Pupil...Teacher...

School...

Name and author of textbook used ....

Your answers to Part I should be recorded on this answer sheet.

Part I

Answer all questions in this part.

1...............................	11..............................	21..............................
2...............................	12..............................	22..............................
3...............................	13..............................	23..............................
4...............................	14..............................	24..............................
5...............................	15..............................	25..............................
6...............................	16..............................	26..............................
7...............................	17..............................	27..............................
8...............................	18..............................	28..............................
9...............................	19..............................	29..............................
10..............................	20..............................	30 Answer question 30 on the other side of this sheet.
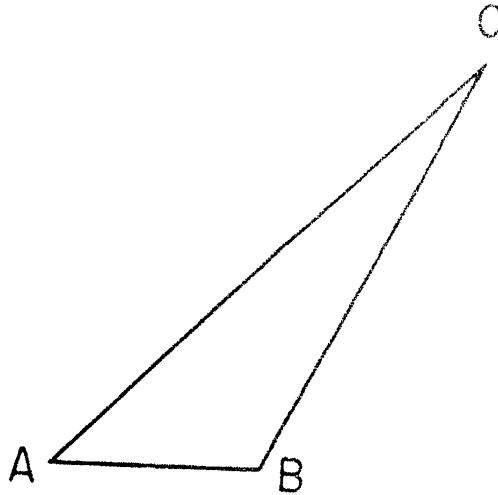
The declaration below should be signed when you have completed the examination.

I do hereby affirm, at the close of this examination, that I had no unlawful knowledge of the questions or answers prior to the examination, and that I have neither given nor received assistance in answering any of the questions during the examination.

_____

Signature

FOR TEACHERS ONLY

10

SCORING KEY

TENTH YEAR MATHEMATICS

Friday, June 18, 1976 — 1:15 to 4:15 p.m., only

Use only *red* ink or *red* pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 20–29, allow credit if the pupil has written the correct answer instead of the numeral 1, 2, 3, or 4.

(1) 20	(11) 3π	(21) 1
(2) 90	(12) 16	(22) 4
(3) 40	(13) (0,0) *or* $\begin{array}{l} x = 0 \\ y = 0 \end{array}$	(23) 2
(4) $\overline{TS}$ *or* TS *or* r	(14) 25	(24) 3
(5) 60	(15) 60	(25) 4
(6) 3π	(16) 4	(26) 1
(7) 900° *or* 900	(17) 6	(27) 3
(8) (2,3) *or* $\begin{array}{l} x = 2 \\ y = 3 \end{array}$	(18) 4	(28) 2
(9) 105	(19) $\frac{7}{25}$ *or* .28	(29) 3
(10) 8	(20) 1	

[OVER]

Part II

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics*. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(33) b (1) $\frac{3}{4}$ [2]

     (2) $\frac{9}{16}$ [2]

(35) a 13   [7]

     b $\sqrt{34}$ [3]

(36) (1) 70   [2]

     (2) 60   [2]

     (3) 65   [2]

     (4) 30   [2]

     (5) 150   [2]

(37) a $-\frac{2}{3}$ [2]

     b $-\dfrac{b+5}{3}$ [3]

     c -3 [2]

     d $y - 1 = \frac{3}{2}(x - 1)$
         OR   [3]
      $2y = 3x - 1$