The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

TENTH YEAR MATHEMATICS

Monday, June 17, 1963 — 1:15 to 4:15 p.m., only

Name of pupil..... Name of school.....

Name and author of textbook used.....

Name of teacher.....

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed.

1 A tangent and a secant are drawn to a circle from an external point. The tangent is 6 inches long; the secant is 12 inches long. Find the number of inches in the length of the external segment of the secant. 1.....

2 The number of degrees in angle A is equal to $\frac{1}{5}$ the number of degrees in its complement. Find the number of degrees in angle A. 2.....

3 The area of a rhombus is 20. The length of one diagonal is 10. Find the length of the other diagonal. 3.....

4 If the perimeter of a regular polygon is 40 and its area is 100, find the apothem. 4.....

5 The number of degrees in a pair of vertical angles is represented by x and $3x - 48$. What is the value of x? 5.....

6 An isosceles trapezoid has bases of lengths 10 inches and 14 inches, and the length of each diagonal is 13 inches. Find the number of inches in the height of the trapezoid. 6.....

7 Two secants are drawn to a circle from an external point forming an angle of 90°. If the smaller intercepted arc contains 30 degrees, find the number of degrees in the larger intercepted arc. 7.....

8 The angles of a triangle are in the ratios 1 : 2 : 3. The length of the shortest side of the triangle is 4. Find the length of the longest side. 8.....

9 The hypotenuse of a right triangle is 9 and one side is 3. Find the length of the other side. [Leave answer in radical form.] 9.....

10 Find the area of a sector of 45° in a circle of radius 4 inches. [Leave answer in terms of π.] 10.....

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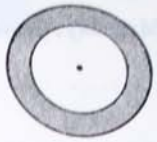
11 In a circle chord CD is bisected at G by chord EF. If CG is 8 and EG is 16, find the length of chord EF. 11.....

12 Find the area of the circular ring formed by two concentric circles, one with a radius of 7 and the other with a radius of 10. [Leave answer in terms of π.]



12.....

13 Find the length of a side of a square whose diagonal is $3\sqrt{2}$. 13.....

14 When the angle of elevation of the sun is 35°, the shadow of a tree on level ground is 150 feet long. Find the height of the tree to the *nearest foot*. 14.....

15 Find the coordinates of the midpoint of the line segment whose end points are A (2,—6) and B (8,4). 15.....

16 What is the length of the line segment joining the points whose coordinates are (7,6) and (—5,1)? 16.....

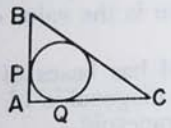
17 The areas of two circles are to each other as 1 : 9. If the radius of the smaller circle is 2, what is the radius of the larger circle? 17.....

18 In an isosceles right triangle the hypotenuse is 18. Find the area of the triangle. 18.....

19 In triangle ABC, angle B is twice as large as angle A. The exterior angle at C is 111°. Find the number of degrees in angle A. 19.....

20 Write an equation of the locus of points whose ordinate is one less than its abscissa. 20.....

21 In the accompanying diagram, triangle ABC is circumscribed about a circle. If P and Q are points of tangency such that $BP = 3$ and $CQ = 5$, find BC.



21.....

Directions (22–28) : Write on the line at the right of *each* of the following the *number* preceding the expression that best completes the statement or answers the question.

22 The lengths of two sides of a triangle are 4 and 5. The third side of the triangle may be

(1) 1
(2) $\frac{1}{2}$
(3) 9
(4) 4

22.....

23 Each interior angle of a regular polygon contains 150°. The number of sides of the polygon is

(1) 12
(2) 10
(3) 8
(4) 6

23.....

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24 Given statement *A*: "The points in a plane at a given distance from a given point lie on a circle." In order to prove that statement *A* satisfies the requirements of a locus theorem, we must prove

(1) statement *A* and its converse
(2) the converse and inverse of statement *A*
(3) only the converse of statement *A*
(4) only statement *A*

24.....

25 Which statement is the best illustration of a good definition?

(1) All right angles are equal.
(2) A square is a rectangle with adjacent sides equal.
(3) A circle is a closed curved line.
(4) An inscribed angle is an angle formed by two chords.

25.....

26 If the midpoints of the sides of a quadrilateral are joined consecutively, the resulting figure is always a

(1) rhombus
(2) rectangle
(3) square
(4) parallelogram

26.....

27 A measurement is given as 9 feet 3 inches. This is understood to mean that the measurement may have any value between

(1) 9 feet 2 inches and 9 feet 4 inches
(2) 9 feet and 10 feet
(3) 9 feet $2\frac{1}{2}$ inches and 9 feet $3\frac{1}{2}$ inches
(4) 8 feet and 10 feet

27.....

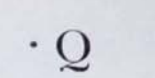
28 Similar polygons are defined as polygons which have

(1) only their corresponding angles equal
(2) both their corresponding angles equal and their corresponding sides equal
(3) only their corresponding sides proportional
(4) both their corresponding sides proportional and their corresponding angles equal

28.....

Directions (29–30): Leave all construction lines on the paper.

29 Construct an angle of 60° having vertex *Q*.



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30 Construct a line parallel to the parallel lines p and q and midway between them.

p

q

Part II

Answer four questions from this part. Show all work unless otherwise directed.

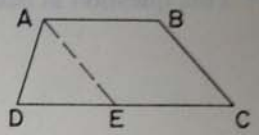
31 Prove *either* *a* or *b* but *not* both: [10]

a An angle formed by a tangent and a secant is measured by one-half the difference of the intercepted arcs.

OR

b The area of a triangle is equal to one-half the product of a side and the altitude drawn to that side.

32 The statements in the accompanying proof are in a logical sequence. On your answer paper, write the numbers 3 to 4, 6 to 9 and 11 to 12 in a vertical column. Next to *each* number, give a reason for the corresponding statement or step in the proof. [It is not necessary to copy the statements of the proof.]



Given: trapezoid $ABCD$ with $AB \parallel DC$ and $BC > AD$

Prove: $\angle DAB < \angle ABC$

Proof

Statements	*Reasons*
1. In trapezoid $ABCD$, $AB \parallel DC$.	1. Given
2. Through A draw $AE \parallel BC$ meeting DC at E.	2. A line can be constructed parallel to a given line through a given point; two lines intersect in a point.
3. $\therefore$ $ABCE$ is a parallelogram.	3. [1]
4. $\therefore$ $AE = BC$.	4. [1]
5. $BC > AD$.	5. Given
6. $\therefore$ $AE > AD$.	6. [1]
7. In triangle ADE, $\angle ADE > \angle AED$.	7. [1]
8. $\angle ADE + \angle DAB = 180°$.	8. [1]
9. $\angle AED + \angle AEC = 180°$.	9. [2]
10. $\angle ADE + \angle DAB = \angle AED + \angle AEC$.	10. Quantities equal to the same quantity are equal to each other.
11. $\angle DAB < \angle AEC$.	11. [2]
12. $\angle AEC = \angle ABC$.	12. [1]
13. $\therefore$ $\angle DAB < \angle ABC$.	13. Same as reason 6

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33 In isosceles triangle DEF, $DE = EF$. Points P and Q are chosen on ED and EF, respectively, so that $EP = EQ$. DQ and FP are drawn intersecting at Z. Prove $\triangle PDF \cong \triangle QFD$. [10]

34 The vertices of a triangle are A (1,0), B (4,7) and C (6,—2).
- *a* Using graph paper, draw triangle ABC. [1]
- *b* Find the area of triangle ABC. [7]
- *c* If the median AD is drawn on side BC, find the coordinates of D. [2]

35 A regular decagon (10-sided polygon) is inscribed in a circle. If the radius of the circle is 4, find to the *nearest integer* the area of the decagon. [10]

36 Given right triangle ABC with hypotenuse AB and altitude CD. $AD = 5$ and $CB = 6$. Answer *a*, *b* and *c*: [Leave irrational answers in radical form.]
- *a* Represent DB by x and write an equation that can be used to find x. [4]
- *b* Solve for x the equation written in answer to *a*. [3]
- *c* Find the length of CD. [3]

*37 Given points P (—2,3), Q (2,3) and O (0,0).
- *a* What is the slope of PQ? [2]
- *b* Write an equation of the straight line which is parallel to PQ and which passes through the point (4,1). [2]
- *c* Write an equation of the locus of points (x,y) which are equidistant from P and Q. [2]
- *d* Write an equation of the line passing through O and Q. [2]
- *e* Write an equation of the line passing through O and P. [2]

* This question is based on one of the optional topics in the syllabus.

FOR TEACHERS ONLY

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SCORING KEY
TENTH YEAR MATHEMATICS

Monday, June 17, 1963 — 1:15 to 4:15 p.m., only

Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 22–28, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3 or 4.

(1) 3

(2) 15

(3) 4

(4) 5

(5) 24

(6) 5

(7) 210

(8) 8

(9) $6\sqrt{2}$ *or* $\sqrt{72}$

(10) 2π

(11) 20

(12) 51π

(13) 3

(14) 105

(15) (5, —1)

(16) 13

(17) 6

(18) 81

(19) 37

(20) $y = x - 1$

(21) 8

(22) 4

(23) 1

(24) 1

(25) 2

(26) 4

(27) 3

(28) 4

[OVER]

Part II

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics*. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(32) 3. If both pairs of opposite sides of a quadrilateral are parallel, the figure is a parallelogram. [1]

4. The opposite sides of a parallelogram are equal. [1]

6. Substitution [1]

7. If two sides of a triangle are unequal, the angles opposite these sides are unequal, and the greater angle lies opposite the greater side. [1]

8. If two parallel lines are cut by a transversal, the interior angles on the same side of the transversal are supplementary. [1]

9. Two adjacent angles whose exterior sides lie in the same straight line are supplementary. [2]

11. If unequal quantities are subtracted from equal quantities, the remainders are unequal in reverse order. [2]

12. The opposite angles of a parallelogram are equal. [1]

(34) *b* $20\frac{1}{2}$ [7]
c $(5, 2\frac{1}{2})$ [2]

(35) 47 [10]

(36) *a* $x(x+5) = 36$ [4]
b 4 [3]
c $2\sqrt{5}$ *or* $\sqrt{20}$ [3]

(37) *a* 0 [2]
b $y = 1$ [2]
c $x = 0$ [2]
d $y = \frac{3}{2}x$ [2]
e $y = -\frac{3}{2}x$ [2]