TENTH YEAR MATHEMATICS

Monday, June 18, 1962 — 1:15 to 4:15 p.m., only

Name of pupil..Name of school..

Name and author of textbook used..

Name of teacher..

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, it is desirable to leave irrational answers in terms of π or in radical form.

1 The coordinates of the end points of a diameter of a circle are $(-2, 4)$ and $(6, -2)$. Find the coordinates of the center of the circle.   1................

2 Find the number of degrees in an interior angle of a regular polygon of 20 sides.   2................

3 In triangle ABC, a line parallel to AC intersects AB at D and CB at E. If $DE = 2$ and $AC = 3$, what is the ratio of the area of triangle DBE to the area of triangle ABC?   3................

4 Find the length of the longest side of the triangle whose vertices are $A(1, 1)$, $B(9, 7)$ and $C(9, 16)$.   4................

5 In circle O at the right, AB is a diameter, angle BOD contains $15°$ and angle EOA contains $85°$. Find the number of degrees in angle ECA.

   5................
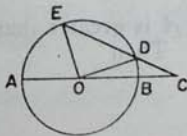
6 Find the area of a square whose diagonal is 10.   6................
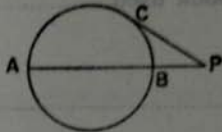
7 The bases of a trapezoid are 6 inches and 10 inches. If its area is 64 square inches, find the number of inches in the length of the altitude.   7................

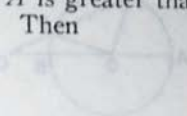8 The angles of a triangle are in the ratios $2:3:7$. Find the number of degrees in the largest angle.   8................

9 In circle O, chords AB and CD intersect at E. $AE = 8$, $EB = 9$, $CE = x$ and $ED = 2x$. Find the value of x.   9................

10 An altitude of an equilateral triangle is $3\sqrt{3}$. Find the length of a side of the triangle.   10................

[1]   [OVER]

11 In triangle *ABC*, angle $B = 56°$ and angle $C = 90°$. If $BC = 16$, find *AC* to the *nearest integer*.

11.................

12 The area of a regular polygon is 25 square inches and the length of its apothem is 2.5 inches. Find the number of inches in the perimeter of the polygon.

12.................

13 Write an equation of the locus of points equidistant from the points $(-1, -1)$ and $(-1, 5)$.

13.................

14 In rectangle *ABCD*, diagonal $AC = 12$ and $BC = 7$. Find the length of *AB*.

14.................

15 Find the area of a regular hexagon whose side is 6.

15.................

16 In the circle shown, *AB* is a diameter. If secant $AP = 8$ and tangent $CP = 4$, find the number of units in the diameter of the circle.

16.................

17 Write an equation of the locus of points whose ordinates are 3 less than half their abscissas.

17.................

18 In a circle of diameter 20 inches, find the number of inches in the length of an arc of 72°.

18.................

19 The altitude drawn to the hypotenuse of a right triangle is 2 and one of the segments of the hypotenuse is 1. Find the length of the other segment.

19.................

20 The median to the hypotenuse of a right triangle is 4. Find the length of the hypotenuse.

20.................

21 The ratio of the corresponding sides of two similar polygons is $3:2$. What is the ratio of the perimeters of the polygons?

21.................

Directions (22–26): Write on the line at the right of *each* of the following the *number* preceding the expression that best completes the statement or answers the question.

22 In triangle *ABC*, angle *A* is greater than angle *B* and the bisectors of angle *A* and angle *B* meet in *D*. Then

(1) $DB > DA$
(2) $DB = DA$
(3) $DB < DA$
(4) no conclusion concerning the relative lengths of *DB* and *DA* can be drawn from the data given

22......

23 Given: All registered voters have reached their 21st birthdays. Which statement expresses a conclusion that follows logically from the given statement?

(1) All persons who have reached their 21st birthdays are registered voters.
(2) If a person has not reached his 21st birthday, then he is not a registered voter.
(3) If a person is not a registered voter, then he has not reached his 21st birthday.
(4) If a person has reached his 21st birthday, then he is a registered voter.

23......

[2]

24 Parallel lines *m* and *n* are 10 inches apart. Point C lies between *m* and *n* at a distance of 3 inches from *m*. The number of points which are equidistant from *m* and *n* and also 7 inches from C is   (1) 1   (2) 2   (3) 3   (4) 4   24......

25 The diagonal of a rectangle is 5. The area of the rectangle

(1) must be 12
(2) must be 12.5
(3) must be 25
(4) cannot be determined from the data given

25......

26 The accompanying diagram shows the construction of an angle bisector. Which proposition is used in the proof of this construction to show that △ ABD ≅ △ BCD?

(1) Two triangles are congruent if two sides and the included angle of one are equal to the corresponding parts of the other.

(2) Two right triangles are congruent if the hypotenuse and a leg of one are equal to the corresponding parts of the other.

(3) Two triangles are congruent if the three sides of one are equal to the three sides of the other.

(4) Two triangles are congruent if two angles and the included side of one are equal to the corresponding parts of the other.
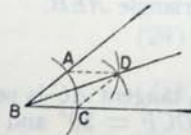
26......

Directions (27–29): If the blank space in each statement is replaced by the word *always*, *sometimes* (but not always) or *never*, the resulting statement will be true. Select the word that will correctly complete *each* statement and write this word on the line at the right.

27 If the diagonals of a parallelogram are unequal and perpendicular, the parallelogram is ... a square.   27................

28 If two lines are cut by a transversal, then the bisectors of a pair of interior angles on the same side of the transversal are ... perpendicular.   28................

29 The altitude and median are drawn from the same vertex of a scalene triangle. The median is ... greater than the altitude.   29................

Directions (30): Leave all construction lines on the paper.

30 Construct a line through P tangent to circle O.
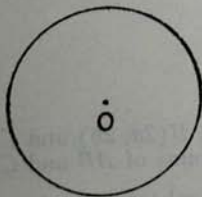
[3]   [OVER]

Part II

Answer four questions from this part. Show all work unless otherwise directed.

31 Prove *either* a or b but *not* both: [10]

 a If two sides of a triangle are equal, the angles opposite these sides are equal.

OR

 b The square of the hypotenuse of a right triangle is equal to the sum of the squares of the legs.

32 Given isosceles trapezoid $ABCD$ as shown with $DC = 20$ and $AD = CB = 13$. Sides AD and BC are extended, meeting in E, and forming isosceles triangles AEB and DEC. The ratio of the area of triangle DEC to the area of triangle AEB is $4:9$.

 a Find the length of DE. [4]
 b Find the length of the altitude drawn from E to AB in triangle AEB. [3]
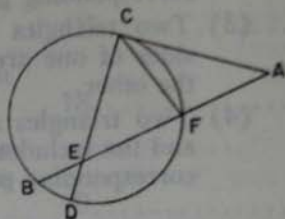 c Find the area of triangle AEB. [3]
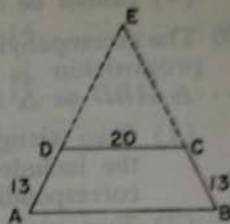
33 In the figure shown, tangent AC is perpendicular to chord CD. Arc $CB = 150°$, angle $DCF = 50°$ and $CE = 5$.

 a Find the number of degrees in arc CF. [4]
 b Find the number of degrees in angle CAB. [2]
 c Find the length of CA to the *nearest integer*. [4]

34 In triangle ABC, $AC = BC$. AC is extended through C to point E, and BC is extended through C to point D so that DE is parallel to AB. Lines DA and EB are drawn. Prove: $\triangle ABD \cong \triangle BAE$ [10]

35 Point $P(2, 3)$ is the center of a circle.

 a A and B are the end points of a diameter of this circle. If the coordinates of A are $(7, 3)$, find the coordinates of B. [3]
 b Using coordinate geometry, show that point $C(-1, 7)$ is a point on the circle. [3]
 c Using coordinate geometry, show that triangle ABC is a right triangle. [4]

36 Given: Two concentric circles with radii x and $3x$, respectively, and line m tangent to the smaller circle.

 a Describe *fully* the locus of points equidistant from the two circles. [2]
 b Describe *fully* the locus of points at a given distance d from line m. [2]
 c How many points are there which satisfy the conditions given in both a and b if
   (1) $d < x$? [2]
   (2) $d = x$? [2]
   (3) $d = 3x$? [2]

*37 Given triangle ABC with vertices $A(0, 0)$, $B(2a, 2b)$ and $C(2c, 2d)$. Use coordinate geometry to show that the line joining the midpoints of AB and CB is parallel to AC. [10]

*This question is based on one of the optional topics in the syllabus.

[4]

INSTRUCTIONS FOR RATING
TENTH YEAR MATHEMATICS

Monday, June 18, 1962 — 1:15 to 4:15 p.m., only

Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 22–26, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3 or 4.

(1) (2,1)	(11) 24	(24) 2
(2) 162	(12) 20	(25) 4
(3) 4:9	(13) $y = 2$	(26) 3
(4) 17	(14) $\sqrt{95}$ or 9.7	(27) never
(5) 35	(15) $54\sqrt{3}$ or 93.4	(28) sometimes
(6) 50	(16) 6	(29) always
(7) 8	(17) $y = \frac{1}{2}x - 3$	
(8) 105	(18) 4π or 12.6	
(9) 6	(19) 4	
(10) 6	(20) 8	
	(21) 3:2	
	(22) 1	
	(23) 2	

Part II

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics.* Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(32)   *a*   26   [4]
      b   36   [3]
      c   540   [3]

(33)   *a*   80   [4]
      b   35   [2]
      c   7   [4]

(35)   *a*   (—3, 3)   [3]

(36)   *a*   A circle with a radius of $2x$ and with the same center as the two given circles   [2]
      b   A pair of lines parallel to m, one on either side of m and at a distance d from m [2]
      c   (1)   4   [2]
          (2)   3   [2]
          (3)   1   [2]