The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

TENTH YEAR MATHEMATICS

Monday, June 19, 1961 — 1:15 to 4:15 p.m., only

Name of pupil..... Name of school.....

Name and author of textbook used.....

Name of teacher.....

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form.

1 Find the area of an equilateral triangle whose side is 8. 1.....

2 Find the area of a right triangle whose legs are 4 and 5. 2.....

3 The number of degrees in a base angle of an isosceles triangle is twice the number of degrees in the vertex angle. Find the number of degrees in the vertex angle. 3.....

4 In triangle ABC, $AB = AC = 13$ inches and $BC = 10$ inches. Find the number of inches in the altitude drawn from A. 4.....

5 In circle O, central angle AOB is equal to 50°. If point C is on minor arc AB, what is the number of degrees in angle ACB? 5.....

6 From point A outside circle O, two secants ABC and ADE are drawn. The number of degrees in angle A is 25 and the number of degrees in arc CE is 80. Find the number of degrees in arc BD. 6.....

7 From point B outside circle O, tangent BC and secant BDE are drawn. If $BD = 4$ and $DE = 5$, what is the length of BC? 7.....

8 In triangle ABC, D is a point on AB and E is a point on AC so that DE is parallel to BC. If $AD = 1$ and $DB = 2$, what is the ratio of DE to BC? 8.....

9 The sides of two regular hexagons are, respectively, 1 and 4. Find the ratio of the area of the smaller hexagon to that of the larger. 9.....

10 The area of a trapezoid is 48 and its altitude is 6. The longer base is three times the shorter base. Find the length of the shorter base. 10.....

11 The circumference of a circle is 14π. Find the length of the radius. 11.....

[OVER]

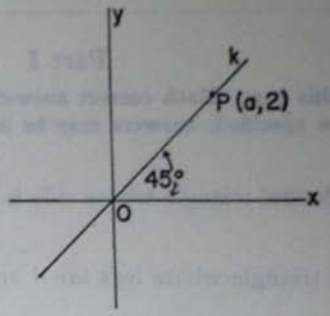
12 The area of circle O is 18 square inches. If there are 40 degrees in central angle AOB, what is the number of square inches in the area of sector AOB? 12.....

13 Find to the *nearest degree* the angle of elevation of the sun if a vertical rod 6 feet long casts a horizontal shadow 4 feet long. 13.....

14 The coordinates of the center of a circle are $(0, 2)$. The circle passes through the point whose coordinates are $(4, 1)$. Find the length of the radius of the circle. 14.....

15 Find the coordinates of the midpoint of the line segment joining the points whose coordinates are $(2, 3)$ and $(5, -1)$. 15.....

16 Point $P\ (a, 2)$ lies on line k, as shown. What is the value of a?



16.....

17 In triangle ABC, angle $A = 30°$, angle $B = 60°$ and $BC = 4$. Find the length of AC. 17.....

Directions (18–26): Write on the line at the right of *each* of the following the *number* preceding the expression that best completes the statement.

18 In triangle ABC, angle C is a right angle and CD is the altitude drawn to AB. If $AD = m$ and $DB = n$, CD is equal to

(1) $m + n$
(2) mn
(3) $\sqrt{m+n}$
(4) $\sqrt{mn}$

18.....

19 If s represents the side of a square and b and h represent the base and altitude of a rectangle, and if the area of the square is equal to the area of the rectangle, then h equals

(1) $\frac{s^2}{b}$
(2) $s^2 - b$
(3) $\frac{b}{s}$
(4) $\frac{b}{s^2}$

19.....

20 Two sides of a triangle are 1 and 2. The third side may be

(1) 1
(2) 2
(3) 3
(4) 4

20.....

21 The area of a square whose diagonal is 8 is

(1) 16 (3) $16\sqrt{2}$
(2) 32 (4) $32\sqrt{2}$ 21.....

22 The smallest number of degrees that an interior angle of a regular polygon may contain is

(1) 30 (3) 60
(2) 45 (4) 90 22.....

23 A quadrilateral must be a parallelogram if

(1) a diagonal divides the quadrilateral into two congruent triangles
(2) the diagonals are equal
(3) the diagonals are perpendicular to each other
(4) the diagonals bisect each other 23.....

24 The locus of points in a plane which are equidistant from two fixed points consists of

(1) one straight line (3) one circle
(2) two straight lines (4) two circles 24.....

25 Given the following statement:

"An exterior angle of a triangle is the angle at the vertex formed by one side of the triangle and an adjacent side produced."

This statement is classified as a

(1) postulate (3) theorem
(2) definition (4) corollary 25.....

26 To prove that an angle is an acute angle, it is sufficient to prove that the angle

(1) is not a right angle
(2) is not an obtuse angle
(3) is neither a right nor an obtuse angle
(4) is greater than 0 degrees and less than 90 degrees 26.....

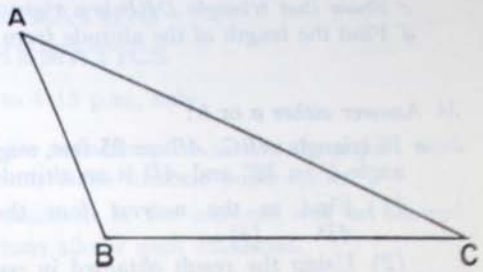
Directions (27–29): If the blank space in each statement is replaced by the word *always*, *sometimes* (but not always) or *never*, the resulting statement will be true. Select the word that will correctly complete *each* statement and write this word on the line at the right.

27 Two triangles are ... congruent if three angles of one are equal to three angles of the other. 27.....

28 If a statement is true, its converse is ... true. 28.....

29 In circle O, the length of chord AB is 12. If AB is bisected by chord CD at point E, then $CE \times ED$ is ... equal to 36. 29.....

30 Find by construction the center of the circle that can be inscribed in triangle ABC. [Leave all construction lines on the paper.]



Part II

Answer four questions from this part. Show all work unless otherwise directed.

31 Prove *either* *a* or *b*:

a An angle formed by a tangent and a secant is measured by one-half the difference of the intercepted arcs. [10]

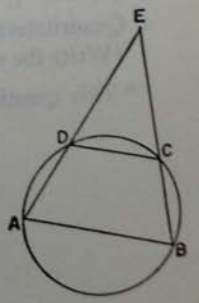
OR

b If in a right triangle the altitude is drawn upon the hypotenuse,

(1) the two triangles thus formed are similar to the given triangle and similar to each other [7] and

(2) each leg of the given triangle is the mean proportional between the hypotenuse and the projection of that leg on the hypotenuse. [3]

32 In the accompanying diagram, $ABCD$ is an inscribed quadrilateral and $AD = BC$. Chords AD and BC are extended to meet at E.
Prove: $AE = BE$. [10]



33 The vertices of a triangle are D (0, 0), E (3, 4) and F (7, 1).

a Using graph paper, draw triangle DEF. [1]

b Find the lengths of the three sides of triangle DEF. [3]

c Show that triangle DEF is a right triangle. [3]

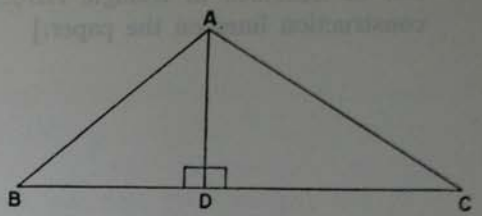
d Find the length of the altitude from E to DF. [3]

34 Answer *either a* or *b*:

a In triangle ABC, $AB = 25$ feet, angle $B = 40°$, angle $C = 32°$ and AD is an altitude.

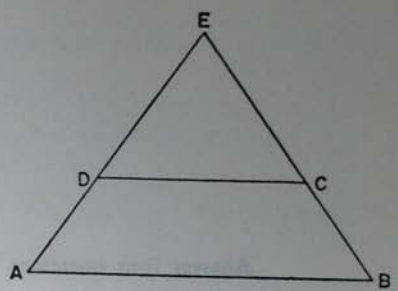
(1) Find to the *nearest foot* the length of AD. [4]

(2) Using the result obtained in part (1), find to the *nearest foot* the length of DC. [6]



OR

b In the accompanying figure, $ABCD$ is an isosceles trapezoid. Bases AB and DC are 30 and 18, respectively, and leg $BC = 10$. AD and BC are extended to meet at E. Find the area of triangle DEC. [10]



35 In parallelogram $ABCD$, $AB > BC$ and diagonal AC is drawn.
Prove: Angle $DAC >$ angle CAB. [10]

*36 The vertices of quadrilateral $MNPQ$ are M (0, 0), N ($2a$, 0), P ($2b$, $2c$) and Q ($2d$, $2e$). Let R, S, T and U be the midpoints of MN, NP, PQ and QM, respectively.

a Using the coordinates of M, N, P and Q, express the coordinates of R, S, T and U. [4]

b By means of slopes show that

(1) $RS \parallel TU$ [2]

(2) $RU \parallel ST$ [2]

c Quadrilateral $RSTU$ must be a (1) rectangle (2) parallelogram (3) rhombus (4) square. [Write the *number* preceding the correct answer on your answer paper after the letter *c*.] [2]

* This question is based on an optional topic in the syllabus.

FOR TEACHERS ONLY

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INSTRUCTIONS FOR RATING
TENTH YEAR MATHEMATICS

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Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 18–26, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3 or 4.

(1) $16\sqrt{3}$ *or* 27.7

(2) 10

(3) 36

(4) 12

(5) 155

(6) 30

(7) 6

(8) 1:3

(9) 1:16

(10) 4

(11) 7

(12) 2

(13) 56

(14) $\sqrt{17}$ *or* 4.1

(15) $(3\frac{1}{2}, 1)$

(16) 2

(17) $4\sqrt{3}$ *or* 6.9

(18) 4

(19) 1

(20) 2

(21) 2

(22) 3

(23) 4

(24) 1

(25) 2

(26) 4

(27) sometimes

(28) sometimes

(29) always

[OVER]

Please refer to the Department's pamphlet Suggestions on the Rating of Regents Examination Papers in Mathematics. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

Part II

(33) *b* 5, 5, $5\sqrt{2}$ [3]

d $\frac{5\sqrt{2}}{2}$ [3]

(34) *a* (1) 16 [4]
(2) 26 [6]

b 108 [10]

(36) *a* $R\ (a, 0)$
$S\ (a + b, c)$
$T\ (b + d, c + e)$ [4]
$U\ (d, e)$

c 2 [2]