TENTH YEAR MATHEMATICS—JUNE 1958 (1)

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form.

1. In parallelogram $ABCD$, angle A is 35°. Find the number of degrees in angle B.
2. Find the area of an equilateral triangle whose side is 2.
3. Two sides of a parallelogram are 6 and 10. If the included angle is 30°, find the area of the parallelogram.
4. A base angle of an isosceles triangle is twice the vertex angle. Find the number of degrees in the vertex angle.
5. A chord 8 inches long is 3 inches from the center of a circle. Find the number of inches in the length of the radius of the circle.
6. The diagonals of a rhombus are $3x$ and $4x$. If the area of the rhombus is 96, find x.
7. Diameter DC of a circle is perpendicular to a chord AB. If arc $AC = 50°$, find the number of degrees in arc BD.
8. Given points A (8, −3) and B (6, 5). Find the coordinates of the midpoint of AB.
9. Write an equation of the locus of points whose ordinates are 2 more than three times their abscissas.
10. The ratio of the corresponding sides of two similar triangles is 2:3. Find the ratio of the area of the smaller triangle to the area of the larger triangle.
11. The sides of a triangle are 6, 8 and 10. Find the smallest side of a similar triangle whose perimeter is 16.
12. The angle of a sector of a circle is 40° and the area of the circle is 81π. Find the area of the sector.
13. In right triangle ABC, angle $C = 90°$, angle $B = 54°$ and $BC = 8$. Find AC to the *nearest integer*.
14. A regular hexagon is inscribed in a circle whose radius is 4. Find the perimeter of the hexagon.
15. The sum of the angles of a polygon is 1440°. Find the number of sides in the polygon.
16. Find the radius of a circle whose center is at (0, 0) and which passes through the point (5, −12).
17. Two tangents to a circle from a point outside intercept a minor arc of 110°. Find the number of degrees in the angle formed by the tangents.
18. In circle O, chords AB and CD intersect at E. If $AE = 3$, $EB = 4$ and $EC = 6$, find DE.

Directions (19-24): Indicate the correct completion for *each* of the following by writing the letter *a*, *b*, or *c* on the line at the right.

19. John proved that if two straight lines are crossed by a transversal making a pair of alternate interior angles equal, the lines are parallel. He said, "If that is true, then it is also true that if two parallel lines are crossed by a transversal, the alternate interior angles are equal." He was (*a*) making a generalization (*b*) reasoning from a converse (*c*) using circular reasoning

20. Given the following theorems:
   (1) An angle inscribed in a circle is measured by one-half the intercepted arc.
   (2) An angle formed by two chords intersecting within a circle is measured by one-half the sum of the intercepted arcs.
   (3) An exterior angle of a triangle is equal to the sum of the two nonadjacent (remote) interior angles of the triangle.

   A logical sequence in which the theorems above can be proved is (*a*) 1, 2, 3 (*b*) 2, 3, 1 (*c*) 3, 1, 2

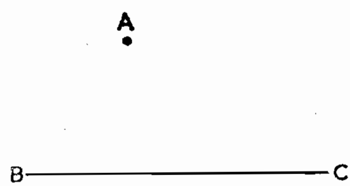
21. Two exterior angles of a triangle may both be (*a*) acute (*b*) right (*c*) obtuse

22. A circle can be circumscribed about any polygon that is (*a*) equilateral (*b*) equiangular (*c*) regular

23. In a circle, an inscribed angle and a central angle intercept the same arc. The ratio of the number of degrees in the inscribed angle to the number of degrees in the central angle is (*a*) 1:2 (*b*) 1:1 (*c*) 2:1

24. The triangle *ABC* is an acute scalene triangle. The shortest line from *A* to *BC* is (*a*) *AC* (*b*) the altitude to *BC* (*c*) the median to *BC*

25. Given point *A* and line *BC*, construct a line from *A* perpendicular to *BC*.



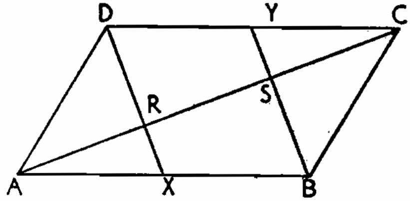
Part II

Answer three questions from this part.

26. Prove: An angle formed by a tangent and a secant is measured by one-half the difference of the intercepted arcs. [10]

27. In the figure, AC is a diagonal of parallelogram $ABCD$. A perpendicular line from D to AC intersects AC at R and is extended to meet AB at X. A perpendicular line from B to AC intersects AC at S and is extended to meet DC at Y. Prove:

a. Triangle ARD is congruent to triangle BSC [5]
b. $AX = YC$ [5]



28. In isosceles triangle ABC, D and E are the midpoints of the equal sides AB and AC, respectively. Prove:

a. $2AB > BC$ [5]
b. $AB > DE$ [5]

29. Prove: If, in a right triangle, the altitude is drawn upon the hypotenuse, (*a*) one triangle thus formed is similar to the given triangle and (*b*) one leg of the given triangle is the mean proportional between the hypotenuse and the projection of that leg on the hypotenuse. [6, 4]

30. Draw two lines AB and CD which intersect at E. Take P, a point on EB.

a. Construct the locus of points equidistant from AB and CD. [4]
b. Construct the locus of points at a fixed distance k from P, where $k > EP$. [2]
c. Indicate by Q, R, etc., all the points that satisfy *both* conditions given in *a* and *b*. [2]
d. If k is equal to EP, the number of different points which are at a distance k from P and also equidistant from AB and CD is (1) 1 (2) 0 (3) 3 [2]

*31. The vertices of quadrilateral $ABCD$ are $A(-5, -5)$, $B(15, 5)$, $C(4, 5)$ and $D(-2, 2)$.

a. Prove by means of slopes that DC is parallel to AB. [7]
b. Write an equation of line BC. [3]

* This question is based on one of the optional topics in the syllabus and may be used in place of any question in *either* part II or part III.

Part III

Answer two questions from this part. Show all work.

32. In trapezoid $ABCD$, AB is the longer base and DC is the shorter base. Angle A is 67° and angle B is 90°. DC is 12.0 and the leg adjacent to angle A is 8.0.

a. Find, to the *nearest tenth*, the altitude of the trapezoid. [3]

b. Find, to the *nearest integer*, the longer base of the trapezoid. [5]

c. Using the answers found in parts *a* and *b*, find, to the *nearest integer*, the area of the trapezoid. [2]

33. At A on circle O, tangent PA is drawn. PCD is a secant which passes through O. $PA = x$, $PC = 2$ and $PD = x + 4$.

a. Write an equation that can be used to solve for x. [2]

b. Find the length of the tangent. [5]

c. Find the radius of the circle. [3]

34. *a.* Find the length of an arc of 45° on a circle whose radius is 8. [2]

b. On another circle, an arc of 60° has the same length as the arc of 45° found in part *a*. Find the radius of this circle. [5]

c. Find the ratio of the area of the smaller circle to the area of the larger circle. [3]

35. *a.* Given the vertices $R\,(9, 10)$, $S\,(13, 2)$, $T\,(-3, -6)$, draw triangle RST. [2]

b. Show that $\triangle\, RST$ is a right triangle. [5]

c. Find the coordinates of the center of the circle which circumscribes $\triangle\, RST$. [3]