a. If DE is represented by x, express BD in terms of x. [1]

b. Which of the following equations can be used to find the length of DE? [3]

(1) $2x + 6 = 8$

(2) $x^2 + 6x = 16$

(3) $x^2 + 6x = 8$

c. Find the length of DE. [3]

d. Find the circumference of the circle. [Answer may be left in terms of π.] [3]

35. Given quadrilateral $ABCD$ whose vertices are A (0, 0), B (6, 8), C (16, 8) and D (10, 0).

a. Using graph paper, construct quadrilateral $ABCD$. [2]

b. If R is the midpoint of AB, S the midpoint of BC and T the midpoint of AD,

(1) find the length of RS [2]

(2) find the length of ST [2]

(3) find the length of RT [2]

c. Show that RST is a right triangle. [2]

June, 1957

PART I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form.

1. The sum of the interior angles of a polygon is 1,980°. Find the number of sides of the polygon. 1.....

2. An exterior angle at the base of an isosceles triangle is 105°. Find the number of degrees in the vertex angle of this triangle. 2.....

3. Chords AB and CD of a circle intersect at E. If AE is 5, EB is 6 and CE is 4, find ED. 3.....

4. In parallelogram $ABCD$, angle A contains x degrees and angle B contains $(2x - 30)$ degrees. Find the number of degrees in angle A. 4.....

5. Secants ABC and ADE are drawn from external point A to a circle. If arc BD is 30° and arc CE is 100°, find the number of degrees in angle A. 5.....

6. The perimeter of a triangle is 18. Find the perimeter of the triangle formed by joining the midpoints of the sides of the given triangle. 6.....

7. A tangent and a secant are drawn to a circle from an external point. If the secant is 9 and its external segment is 4, find the tangent. 7.....

8. In right triangle ABC, CD is the altitude to the hypotenuse AB. If AD is 7 and AB is 10, find AC. 8.....

9. In rectangle $ABCD$, diagonal AC is 11 and side AB is 9. Find angle CAB to the *nearest degree*. 9.....

10. The side of a square is a. Express the diagonal in terms of a. 10.....

11. Find an altitude of an equilateral triangle whose side is 6. 11.....

12. The area of a rhombus is 60 and one diagonal is 10. Find the other diagonal. 12.....

13. The areas of two similar triangles are 20 and 45. If a side of the smaller triangle is 4, find the corresponding side of the larger triangle. 13.....

14. The hypotenuse of a right triangle is 6 inches. Find the number of inches in the median to the hypotenuse. 14.....

15. The circumference of a circle is 54 inches. Find the number of inches in an arc of 80° on this circle. 15.....

16. The point (2, 4) is on the circle whose center is (6, 1). Find the radius of the circle. 16.....

17. The coordinates of A and B are $(-2, 3)$ and (6, 9). Find the coordinates of the midpoint of line segment AB. 17.....

Directions (18–20): Indicate the correct completion for *each* of the following by writing the letter *a*, *b* or *c* on the line at the right.

18. The locus of points equally distant from the points (1, 2) and (1, 8) is the line whose equation is
(*a*) $y = 5$ (*b*) $x = 5$ (*c*) $x + y = 5$ 18.....

19. In triangle ABC, angle A is 60° and angle C is larger than angle B. The longest side of triangle ABC is (*a*) AB (*b*) AC (*c*) BC 19.....

20. If a pupil in a certain school has room 222 as a homeroom, he is a sophomore. Which of the following statements expresses a conclusion that follows logically from the given statement? (*a*) John is a sophomore in this school; therefore he has room 222 as a homeroom. (*b*) Tom is a junior in this school; therefore he does not have room 222 as a homeroom. (*c*) Paul has room 224 as a homeroom in this school; therefore he is not a sophomore. 20.....

Directions (21–24): For *each* of the following, tell whether the statement is always true, sometimes true or never true by writing the word *always*, *sometimes* or *never* on the line at the right.

21. Two parallel lines are cut by a transversal. The bisectors of the two interior angles on the same side of the transversal are perpendicular to each other. 21.....

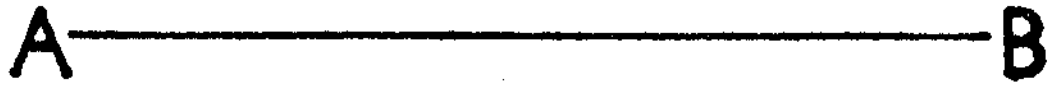
22. The bisectors of two opposite angles of a parallelogram coincide. 22.....

23. If two angles intercept the same arc of a circle, they are equal. 23.....

24. If perpendiculars from the center of a circle to the sides of an inscribed polygon are equal, the polygon is equilateral. 24.....

Directions (25): Leave all construction lines on your paper.

25. Divide line segment AB into three equal parts.



PART II

Answer three questions from this part.

26. Prove: A diameter perpendicular to a chord of a circle bisects the chord and its minor arc. [10]

27. In isosceles triangle ABC, CA equals CB. D is a point on CA and E is a point on CB such that AD equals BE. BD is drawn and extended its own length through D to X, AE is drawn and extended its own length through E to Y, and XA and YB are drawn. Prove:

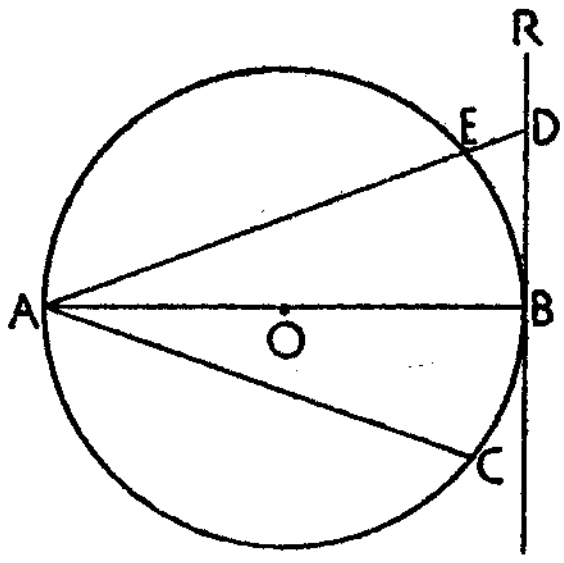
(*a*) $BD = AE$ [6]

(*b*) $XA = YB$ [4]

28. Prove: The area of a triangle is equal to one-half the product of a side and the altitude drawn to that side. [10]

29. AB is a diameter of circle O, RB is a tangent at B, chords AE and AC are drawn on opposite sides of AB such that arc BE equals arc BC and chord AE is extended to meet RB at D.

Prove: $AD:AB = AB:AC$. [10]



30. In quadrilateral $ABCD$, AB equals BC and angle A is greater than angle C. Prove that CD is greater than DA. [Suggestion: draw AC.] [10]

*31. *a* Find the slope of the line k which passes through the points A (1, 2) and B (13, 10). [3]

b If point C (x, 6) lies on the line k, find the value of x. [4]

c Write an equation of the line through the origin and parallel to AB. [3]

PART III

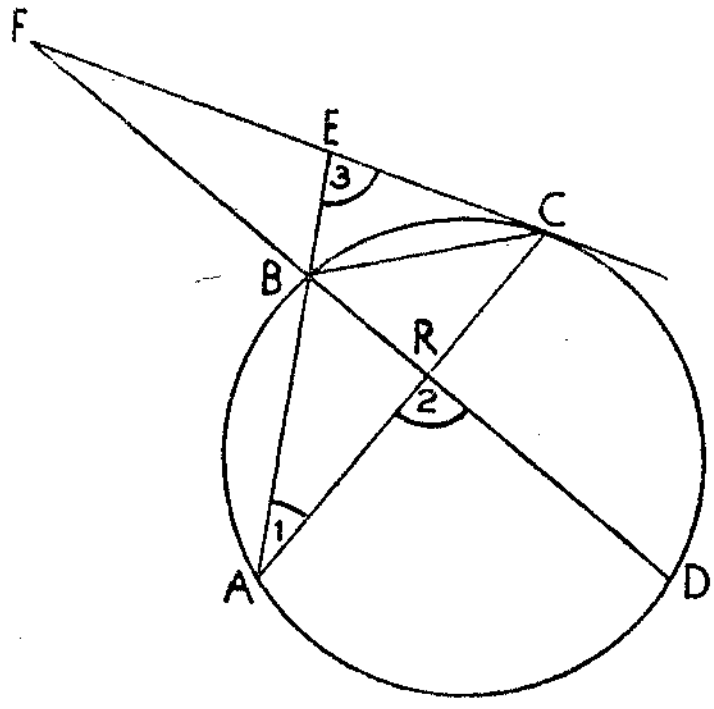
Answer two questions from this part. Show all work.

32. In the accompanying figure, the tangent at C meets chord DB extended at F; chord AB extended meets FC at E; BD intersects AC at R.

Arc AB: arc BC: arc CD: arc DA as 4:3:5:6.

a Find the number of degrees in arcs AB, BC, CD, and DA. [4]

b Find the number of degrees in angles 1, 2, and 3. [6]



*This question is based on one of the optional topics in the syllabus and may be used in place of any question in *either* part II or part III.

33. In isosceles trapezoid $ABCD$, AB is the longer base and DC is the shorter base. AD and BC are extended to meet in R. DC is 6, AB is 18, and AD is 5 more than DR.

a If DR is represented by x, express AD in terms of x. [1]

b Find AD. [4]

c Find the altitude of trapezoid $ABCD$. [3]

d Find the area of trapezoid $ABCD$. [2]

34. The radius of a regular 9-sided polygon is 14.

a Find the apothem of the polygon to the *nearest integer*. [5]

b Find a side of the polygon to the *nearest integer*. [3]

c Using the results found in answer to parts *a* and *b*, find the area of the polygon. [2]

35. The coordinates of the vertices of triangle ABC are A (3, 3), B (14, 1) and C (11, 7).

a Draw triangle ABC on graph paper. [2]

b Show that triangle ABC is a right triangle. [5]

c Find the area of triangle ABC. [3]