b. If the number of degrees in arc DB is represented by $2x$, show that angle CDP and angle DKB are equal. [4]

c. If $AP = 18$ and $KP = 12$, find the radius of the circle. [4]

June, 1956

PART I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form.

1. The hypotenuse of a right triangle is 7 and one leg is 4. Find the other leg.

2. The angles of a triangle are in the ratio 3:5:7. Find the number of degrees in the *smallest* angle of the triangle.

3. Two angles are complementary and one angle is 48° greater than the other. Find the number of degrees in the *smaller* angle.

4. Find the number of degrees in the sum of the interior angles of a polygon of twelve sides.

5. The bases of an isosceles trapezoid are 9 and 15, and each base angle contains 45°. Find the altitude of the trapezoid.

6. Find a side of an equilateral triangle whose area is $16\sqrt{3}$.

7. Find the area of a triangle whose base is 12 and whose altitude is 7.

8. Find the area of a circle whose radius is 3.

9. Find the length of an arc of 70° in a circle whose radius is 9.

10. In right triangle ABC, angle $C = 90°$, $AB = 10$ and $AC = 4$. Find angle A to the *nearest degree.*

11. Corresponding sides of two similar triangles are 2 and 3. If the area of the smaller triangle is 12, find the area of the larger triangle.

12. In right triangle ABC, CD is the altitude on the hypotenuse. If $AB = 10$ and $AC = 7$, find AD.

13. From a point outside a circle a tangent and a secant are drawn. If the secant is 12 and its external segment is 3, find the tangent.

14. Chords AB and CD intersect within a circle at E. If $AE = r$, $EB = s$ and $CE = t$, express ED in terms of r, s and t.

15. Points A (7, 4) and B (−3, 8) are the end points of diameter AB of a circle whose center is O. Find the coordinates of point O.

16. Write an equation of the locus of points whose ordinates are three times their abscissas.

17. Find the distance from point A (6, 5) to point B (1, 7).

18. In quadrilateral $ABCD$, it has been proved that AB is parallel to DC and that BC is parallel to AD. Which of the following statements, a or b, may be used to prove that $ABCD$ is a parallelogram?

a. The opposite sides of a parallelogram are parallel.

b. A parallelogram is a quadrilateral whose opposite sides are parallel.

Directions (19–24): Indicate the correct completion for *each* of the following by writing the letter a, b or c on the line at the right.

19. In circle O, inscribed angle BAC and central angle BOC intercept the same arc BC. If angle $A = 50°$, then angle BOC is equal to (*a*) 25° (*b*) 50° (*c*) 100°

20. In any triangle, the point which is equidistant from the three vertices is the intersection of (*a*) the angle bisectors (*b*) the perpendicular bisectors of the sides (*c*) the medians

21. The locus of the centers of all circles tangent to two parallel lines is (*a*) a point (*b*) a line (*c*) two lines

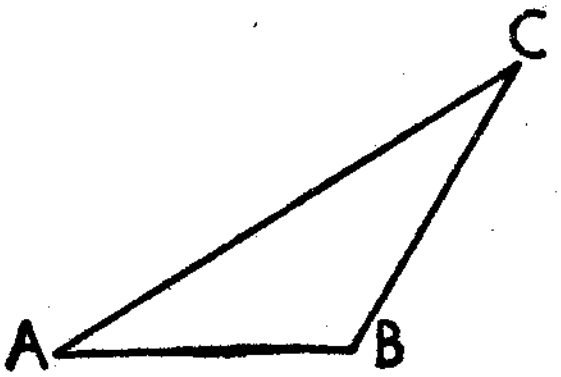
22. A regular octagon has a side s and an apothem a. The area of the octagon is (*a*) $4as$ (*b*) $8as$ (*c*) $16as$

23. The bases of trapezoid $ABCD$ are AD and BC and its diagonals intersect at E. Triangles AEB and CED are always (*a*) congruent (*b*) similar (*c*) equal in area

24. In proving an exercise a pupil says, "In a given triangle ABC, draw median AM, bisecting angle A." Line AM is therefore (*a*) underdetermined (*b*) determined (*c*) overdetermined

Directions (25): Leave all construction lines on the paper.

25. Construct a triangle congruent to triangle ABC.



PART II

Answer three questions from this part.

26. Prove: If two sides of a triangle are equal, the angles opposite these sides are equal. [10]

27. In parallelogram $ABCD$, perpendiculars drawn to diagonal AC from B and D meet AC at points E and K respectively.

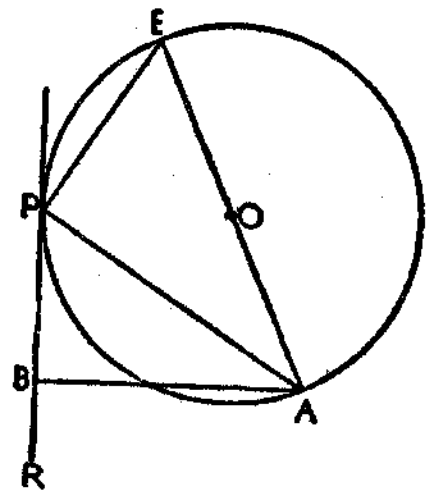
a. Prove: $BE = KD$. [7]

b. A point H is taken on AC, and BH and DH are drawn. Prove that triangle ABH is equal in area to triangle ADH. [3]

28. Prove: The area of a trapezoid is equal to one half the product of the altitude and the sum of the bases. [10]

29. In the figure at the right, PR is a tangent, PA and PE are chords and AE is a diameter of circle O. AB is perpendicular to PR.

Prove: $AB \times AE = (AP)^2$. [10]



30. In triangle ABC, AC is greater than AB. CA is extended through A to a point D, and BD is drawn. Prove that DC is greater than DB. [10]

*31. *A.* (1) Write a formula for the slope m of a straight line in terms of the coordinates (x_1, y_1), (x_2, y_2) of two points on the line. [2]

(2) Find the slope of the straight line which passes through the points $(-1, 3)$ and $(2, -4)$. [2]

B. List the numbers 1–3 on your answer paper. Indicate the correct completion for *each* of the following by writing the letter *a*, *b* or *c* after the number.

(1) The straight line which passes through the points (8, 4) and (2, 4) (*a*) has no slope (*b*) has a slope of zero (*c*) has a slope of 6 [2]

(2) The line whose equation is $y = 2x - 5$ is parallel to the line whose equation is (*a*) $2x + y - 5 = 0$ (*b*) $2x - y + 5 = 0$ (*c*) $x - 2y + 5 = 0$ [2]

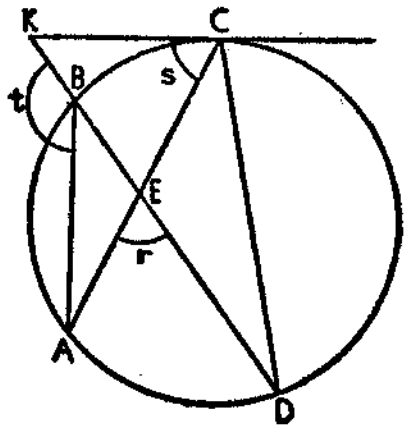
(3) The equation of the straight line which passes through the origin and the point (2, 0) is (*a*) $y = 2$ (*b*) $y = 2x$ (*c*) $y = 0$ [2]

* This question is based on one of the optional topics in the syllabus and may be used in place of any question in *either* part II or part III.

PART III

Answer two questions from this part. Show all work.

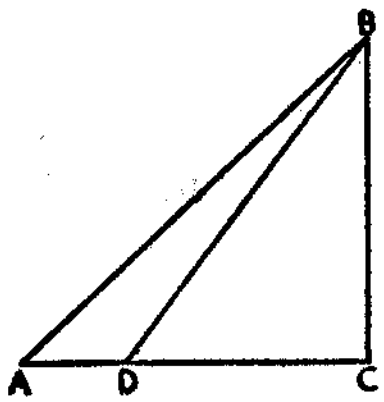
32. In the accompanying figure, AB and CD are chords. Chords AC and DB intersect at E. The tangent at C meets DB extended at K. Arc $AB = 80°$ and arcs BC, CD and AD are represented by $x°$, $(2x - 8)°$ and $(x + 32)°$ respectively.



a. Find the number of degrees in arcs BC, CD and DA. [3, 1, 1]

b. Find the number of degrees in angles r, s, and t. [2, 1, 2]

33. In the figure at the right, B represents the position of a captive balloon connected by a cable to a ground station at A. Point C is on the ground directly below the balloon, and D is an observation point. Points A, D and C lie in a straight line on level ground. Angle $A = 43°$, angle $BDC = 54°$, angle $C = 90°$, and $DC = 170$ yards.



a. Find the height BC of the balloon to the *nearest yard.* [4]

b. Using the result found in part *a*, find the length AB of the cable to the *nearest yard.* [6]

34. Points A, B and C lie on a circle with B the midpoint of the major arc AC. The diameter through B intersects chord AC at D and minor arc AC at E. AC is 8 inches in length, and BD is 6 inches longer than DE.

a. If DE is represented by x, express BD in terms of x. [1]

b. Which of the following equations can be used to find the length of DE? [3]

(1) $2x + 6 = 8$

(2) $x^2 + 6x = 16$

(3) $x^2 + 6x = 8$

c. Find the length of DE. [3]

d. Find the circumference of the circle. [Answer may be left in terms of π.] [3]

35. Given quadrilateral $ABCD$ whose vertices are A (0, 0), B (6, 8), C (16, 8) and D (10, 0).

a. Using graph paper, construct quadrilateral $ABCD$. [2]

b. If R is the midpoint of AB, S the midpoint of BC and T the midpoint of AD,

(1) find the length of RS [2]

(2) find the length of ST [2]

(3) find the length of RT [2]

c. Show that RST is a right triangle. [2]

June, 1957

PART I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form.

1. The sum of the interior angles of a polygon is 1,980°. Find the number of sides of the polygon. 1.....

2. An exterior angle at the base of an isosceles triangle is 105°. Find the number of degrees in the vertex angle of this triangle. 2.....