TENTH YEAR MATHEMATICS—JUNE 1955 (1)

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed.

1. Find the area of a circle whose radius is 7. [Answer may be left in terms of π.]

2. Find the diagonal of a square whose side is 12. [Answer may be left in radical form.]

3. Find the sum of the interior angles of a polygon of nine sides.

4. Two corresponding sides of two simllar triangles are 4 and 6. If an altitude of the smaller triangle is 5, find the corresponding altitude of the larger triangle.

5. A side of a regular polygon of 9 sides is s and the apothem is a. Express the area of the polygon in terms of s and a.

6. A ladder 40 feet long is placed against the side of a building which is on level ground. The ladder makes an angle of 72° with the ground. What vertical distance on the side of the building can the top of the ladder reach? [Answer may be left to the *nearest foot.*]

7. A diameter of a circle is perpendicular to a chord. If the chord is 12 inches long and the shorter segment of the diameter is 3 inches, find the longer segment of the diameter.

8. The ratio of the corresponding sides of two similar polygons is 2:3. Find the ratio of the area of the smaller polygon to the area of the larger polygon.

9. Find the area of an equilateral triangle whose side is 8. [Answer may be left in radical form.]

10. Two tangents to a circle from a point outside the circle intercept an arc of 100°. How many degrees are there in the angle formed by the two tangents?

11. In parallelogram $ABCD$, angle A exceeds angle B by 30°. How many degrees are there in angle B?

12. Side BC of rectangle $ABCD$ is extended through C to point E. If angle $BDE = 90°$, $BC = 4$, and $CE = 9$, find DC.

13. In a circle of radius 18, find the length of an arc of 140°. [Answer may be left in terms of π.]

14. Find the abscissa of the midpoint B of line segment AC if the abscissas of points A and C are 2 and 5 respectively.

15. Given the points $A(-4, 2)$ and $B(4, 8)$. Find the length of line segment AB.

TENTH YEAR MATHEMATICS—JUNE 1955 (2)

Directions (16-19): Indicate whether *each* statement is true or false by writing the word *true* or *false* on the line at the right.

16. As the number of sides of a regular polygon inscribed in a given circle increases, the apothem increases.

17. If in triangle ABC angle $A = 70°$ and angle $B = 30°$, then the longest side of the triangle is BC.

18. If in triangle ABC angle $A = 44°$ and angle $B = 73°$ and in triangle DEF angle $D = 73°$ and angle $E = 63°$, then triangle ABC is similar to triangle DEF.

19. If quadrilateral $ABCD$ is inscribed in a circle, it is possible for angle A to equal 102° when angle C equals 79°.

Directions (20-23): Indicate the correct answer to *each* of the following by writing the letter *a*, *b* or *c* on the line at the right.

20. The locus of points equidistant from the points (5, 0) and (0, 5) is given by the equation (*a*) $y = x$ (*b*) $x + y = 5$ (*c*) $y = 5x$

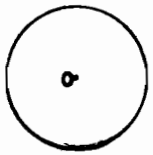
21. Similar triangles are *defined* as triangles in which
   a. the three angles of one triangle are equal to the three angles of the other triangle and the corresponding sides are proportional
   b. the three angles of one triangle are equal to the three angles of the other triangle
   c. the three sides of one triangle are proportional to the three sides of the other triangle

22. Which of the following statements is *not always* true?
   a. The diagonals of a rectangle are equal.
   b. The diagonals of an isosceles trapezoid are equal.
   c. The diagonals of a parallelogram bisect the angles of the parallelogram.

23. John is the strongest boy in the senior class at high school K. Which of the following statements expresses a conclusion that logically follows from the given statement?
   a. If Fred is not so strong as John, Fred is a senior.
   b. Since Dick is a senior at high school K, he is not so strong as John.
   c. Since Tom is a junior at high school K, he is not so strong as John.

Directions (24-25): Leave all construction lines on the paper.

24. Inscribe an equilateral triangle in circle O.



25. Divide line segment AB into three equal parts.

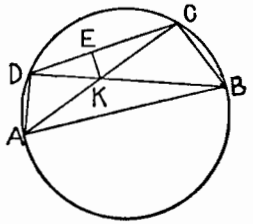
A ——————— B

Part II

Answer three questions from this part.

26. Prove: The sum of the angles of a triangle is equal to a straight angle. [10]

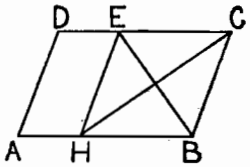
27. In the figure at the right, $ABCD$ is a quadrilateral inscribed in a circle. AB is a diameter and diagonals AC and DB intersect at K. KE is perpendicular to DC.



Prove: $\frac{AB}{DK} = \frac{AC}{DB}$. [10]

28. Prove: The area of a triangle is equal to one half the product of a side and the altitude drawn to that side. [10]

29. In the accompanying figure, $ABCD$ is a parallelogram. The bisector of angle B meets CD in E; the bisector of angle C meets AB in H; and EH is drawn.



a. Prove that $EC = BC$. [5]

b. Prove that $BCEH$ is a parallelogram. [5]

30. AB and CD are straight lines that intersect at Q. P is a point on AB.

a. *Construct* the locus of points equidistant from AB and CD. [4]

b. *Construct* the locus of points at a distance PQ from P. [2]

c. How many points are there equidistant from AB and CD and also at a distance from P equal to PQ? [2]

d. How many points are there equidistant from AB and CD and also at a distance from P greater than PQ? [2]

*31. Given points $A(0, 0)$, $B(8, 0)$ and $C(10, 6)$.

a. Write the equation of the line through A parallel to line BC. [4]

b. Write the equation of the line through C parallel to line AB. [2]

c. The lines whose equations were found in answer to parts *a* and *b* intersect at D. Find the coordinates of D. [2]

d. Find the area of quadrilateral *ABCD*. [2]

• This question is based on one of the optional topics in the syllabus and may be used in place of any question in *either* part II or part III.

Part III

Answer two questions from this part. Show all work.

32. In an isosceles trapezoid the length of the shorter base is x feet.

a. If each leg is 1 foot more than twice the shorter base, express the length of a leg in terms of x. [1]

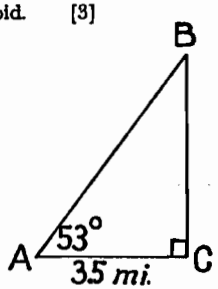
b. If the longer base is 2 feet less than 3 times the shorter base, express the length of the longer base in terms of x. [1]

c. If the perimeter of the trapezoid is 48 feet, find x. [3]

d. Using the value of x found in *c*, find the length of the longer base and a leg of the trapezoid. [2]

e. Find the length of the altitude of the trapezoid. [3]

33. As represented by the diagram at the right, a boat sails on a triangular course from *A* due east to *C* and then due north to *B* and back to *A*. The angle between *AB* and *AC* is 53°. If the distance from *A* to *C* is 3.5 miles,



a. find *BC* to the *nearest tenth* of a mile [3]

b. find *AB* to the *nearest tenth* of a mile [5]

c. find the length of the triangular course to the *nearest mile* [2]

34. A man has a flower garden in the shape of a *segment* of a circle. The arc of the *segment* is 90° and the radius of the circle is 20 feet. [3.14 may be used as a value of π.]

a. Find to the *nearest foot* the number of feet of fencing required to enclose the garden. [5]

b. Find the area of the garden to the *nearest square foot*. [5]

35. Given points $A(4, 3)$, $B(15, 1)$, and $C(12, 9)$.

a. Find the area of triangle *ABC*. [6]

b. Find the length of side *AC*. [2]

c. Using the results found in answer to parts *a* and *b*, find the length of the altitude from point *B* to side *AC*. [2]