The University of the State of New York

321ST HIGH SCHOOL EXAMINATION

TENTH YEAR MATHEMATICS

Monday, June 21, 1954 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in tenth year mathematics, (*c*) author of textbook used.

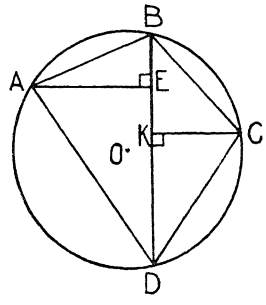
The minimum time requirement is four or five recitations a week for a school year.

Part II

Answer three questions from part II.

26 Prove: An angle formed by two chords intersecting inside a circle is measured by one half the sum of the intercepted arcs. [10]

27 In the figure at the right, quadrilateral $ABCD$ is inscribed in circle O. Perpendiculars are drawn to diagonal BD from A and C meeting BD at E and K respectively. If arc AB is equal to arc BC, prove that $ED \times DC = AD \times KD$. [10]



28 In parallelogram $ABCD$, M is the mid-point of side DC. Line AM extended meets BC extended at K.

a Prove that triangle ADM is congruent to triangle CMK. [4]

b Prove that triangle AKB is equal in area to parallelogram $ABCD$. [6]

29 Prove: The area of a regular polygon is equal to one half the product of its perimeter and its apothem. [10]

30 Given line l, S a point on l, and R a point not on l.

a *Construct* and *label*, or describe *fully*

(1) the locus of the centers of circles passing through R and S [4]

(2) the locus of the centers of circles tangent to line l at S [4]

b How many circles can be drawn tangent to line l at S and passing through R? [2]

*31 The vertices of a quadrilateral $ABCD$ are $A(-1, -3)$, $B(7, -1)$, $C(5, 4)$ and $D(-3, 2)$.

a Using graph paper, plot these vertices and draw the quadrilateral. [4]

b Find the slope of each side of the quadrilateral. [4]

c Show that $ABCD$ is a parallelogram. [2]

* This question is based on one of the optional topics in the syllabus and may be used in place of any question in either part II or part III.

Part III

Answer two questions from part III. All work, including computation, should be shown.

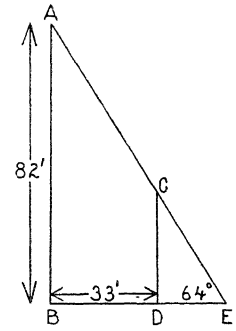
32 Triangle ABC is inscribed in a circle. Arcs AB, BC and CA are represented by $x + 70°$, $2x + 20°$ and $3x - 30°$ respectively.

a Find x. [3]

b Show that triangle ABC is equilateral. [3]

c If the bisector of angle B and the tangent to the circle at C when extended meet in S, find the number of degrees in angle S. [4]

33 In the figure at the right, AB is a tower and CD is a flagpole, both of which are standing on level ground. The height of the tower is 82 feet and the distance (BD) from the foot of the pole to the foot of the tower is 33 feet. If the angle of elevation of the sun (angle AEB) is 64°, find to the *nearest foot*



a the length of the shadow (BE) of the tower [5]

b the height of the pole [5]

34 The difference between the areas of two similar triangles is 18 square feet.

a If A represents the area of the smaller triangle, represent the area of the larger triangle in terms of A. [1]

b If the ratio of the area of the larger triangle to the area of the smaller triangle is 4 : 1, write an equation that can be used to find A. [2]

c Find A. [3]

d If one side of the smaller triangle is 5 feet, find the corresponding side of the larger triangle. [4]

35 The vertices of triangle ABC are $A(3, 2)$, $B(11, 6)$ and $C(9, 10)$. D is the mid-point of AC.

a Find the coordinates of D. [2]

b Find the length of DA, DB, and DC. [6]

c Show that ABC is a right triangle. [2]

Fill in the following lines:

Name of pupil.....Name of school.....

Part I

Answer all questions in part I. Each correct answer will receive 2 credits. No partial credit will be allowed.

1 The radius of a circle is 8. Find the circumference of the circle. [Answer may be left in terms of π.] 1.....

2 A secant and a tangent are drawn to a circle from an external point. If the tangent is 12 and the external segment of the secant is 6, find the secant. 2.....

3 Find the number of degrees in the sum of the interior angles of a polygon of 12 sides. 3.....

4 In right triangle ABC, CD is the altitude upon the hypotenuse. If $AB = 12$ and $DB = 3$, find BC. 4.....

5 The altitude of a trapezoid is 9 and the bases are 6 and 18. Find the area of the trapezoid. 5.....

6 Find the area of a square whose diagonal is 8. 6.....

7 In rectangle $ABCD$, E is the mid-point of DC and F is the mid-point of AD. If $FE = 7$, find diagonal AC. 7.....

8 Find the altitude of an equilateral triangle whose side is 10. [Answer may be left in radical form.] 8.....

9 How many points are there that are 2 inches from a given line and also 3 inches from a given point on the line? 9.....

10 The radii of two circles are in the ratio 1 : 3. Find the circumference of the larger circle if the circumference of the smaller circle is 16. 10.....

11 Chords AB and CD intersect within a circle at E. $AE = EB$, $CE = 4$ and $ED = 9$. Find EB. 11.....

12 The diagonals of a rhombus are 12 and 16. Find a side of the rhombus. 12.....

13 The radius of a circle is 4 and the angle of a sector of the circle is 45°. Find the area of the sector. [Answer may be left in terms of π.] 13.....

14 The approach to a bridge rises 19 feet for every 100 feet of road traveled. Find to the *nearest degree* the angle that the road makes with the horizontal. 14.....

15 The point $(9, -4)$ is on the circle whose center is $(1, 2)$. Find the radius of the circle. 15.....

16 Two parallel lines are cut by a transversal. One of the two interior angles on the same side of the transversal is 30° more than the other. Find the number of degrees in the smaller angle. 16.....

17 Write an equation of the locus of points whose ordinates are five times their abscissas. 17.....

18 If the coordinates of A and B are $(8, 3)$ and $(4, 7)$, find the coordinates of the mid-point of line segment AB. 18.....

Directions (19–21) : For *each* of the following, if the statement is *always* true, write the word *true* on the line at the right; if it is *not always* true, write the word *false*.

19 If two polygons have their corresponding angles equal, the polygons are similar. 19.....

20 If two triangles have equal bases and equal altitudes, the triangles are congruent. 20.....

21 A circle can be circumscribed about any rectangle. 21.....

Directions (22–24) : Indicate the correct completion for *each* of the following by writing the letter *a, b* or *c* on the line at the right.

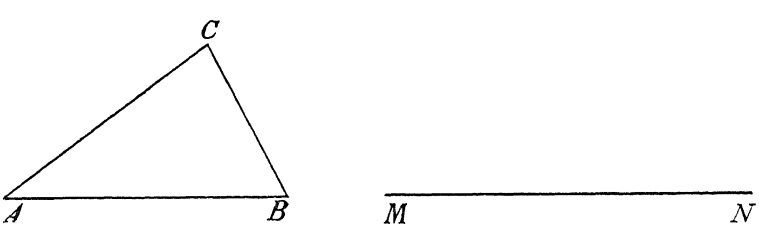
22 All residents of this state who are registered voters are over 21 years of age. If John is a resident of this state, it is correct to conclude that (*a*) if John is over 21, he is a registered voter (*b*) if John is a registered voter, he is over 21 (*c*) if John is not a registered voter, he is not over 21 22.....

23 A student gives the following statement as the definition of a parallelogram: "A parallelogram is a quadrilateral whose opposite sides are parallel." The amount of information contained in this statement as a definition of a parallelogram is (*a*) too little (*b*) just enough (*c*) too much 23.....

24 In triangle ABC, if $AB = BC$ and angle $A = 50°$, (*a*) AC is less than AB (*b*) AC is equal to AB (*c*) AC is greater than AB 24.....

Directions (25) : Leave all construction lines on the paper.

25 On line segment MN corresponding to side AB of triangle ABC, construct a triangle similar to triangle ABC.



FOR TEACHERS ONLY

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INSTRUCTIONS FOR RATING
TENTH YEAR MATHEMATICS

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Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use check marks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 22–24, allow credit if the pupil has written the correct answer instead of the letter *a, b* or *c*.

(1) 16π
(2) 24
(3) 1800
(4) 6
(5) 108
(6) 32
(7) 14
(8) $5\sqrt{3}$
(9) 4
(10) 48
(11) 6
(12) 10
(13) 2π
(14) 11°
(15) 10
(16) 75
(17) $y = 5x$
(18) (6, 5)
(19) false
(20) false
(21) true
(22) *b*
(23) *b*
(24) *c*