The University of the State of New York

315TH HIGH SCHOOL EXAMINATION

TENTH YEAR MATHEMATICS

Monday, June 16, 1952 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in tenth year mathematics, (*c*) author of textbook used.

The minimum time requirement is four or five recitations a week for a school year.

Part II

Answer three questions from part II.

26 Prove that the sum of the angles of a triangle is equal to a straight angle. [10]

27 Prove that a diameter perpendicular to a chord of a circle bisects the chord and its minor arc. [10]

28 BC is the base of isosceles triangle ABC inscribed in a circle. P is any point on BC. AP is drawn and extended to meet the circle in S, and chord SC is drawn.

Prove: (1) $\angle ACP = \angle ASC$ [5]

(2) $AP : AC = AC : AS$ [5]

29 In an acute triangle ABC, the altitude from B meets side AC in D. If angle ABD is greater than angle CBD, prove that AB is greater than BC. [10]

30 Each of the arguments given below is *unsound* for *one* of the following reasons:

(1) A converse was assumed to be true.

(2) An inverse (opposite) was assumed to be true.

(3) Circular reasoning was used.

(4) Indirect reasoning was used incorrectly.

Write the letters *a, b, c, d* and *e* on your answer paper, and for each argument indicate the reason it is unsound by writing 1, 2, 3 or 4.

a Wherever there is a high standard of living there are excellent schools. Therefore, wherever there are excellent schools there will be a high standard of living. [2]

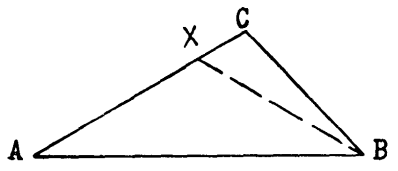
b John has learned that parallel chords intercept equal arcs on a circle. In his diagram, chords AB and CD are intersecting chords. John therefore concludes that arc AC can not equal arc BD. [2]

c A circle can be circumscribed about a regular polygon. Therefore, if a circle can be circumscribed about a polygon, the polygon must be regular. [2]

d Given: $\triangle ABC$ in which $AC > CB$

Prove: $\angle B > \angle A$

Construct angle ABX equal to angle A. Now angle B is greater than angle ABX because the whole is greater than any of its parts. Therefore, angle B is greater than angle A because a quantity may be substituted for its equal. [2]



e Mr. Smith finds that his car is overheating. Some of the causes of overheating are a faulty thermostat, a broken fan belt, a broken water pump and a clogged radiator. With little difficulty he has his thermostat, his fan belt and his water pump checked and finds them to be all right. He concludes that his radiator must be clogged. [2]

Part III

Answer two questions from part III.

31 BA and BC are tangents to circle O at A and C respectively and form an angle of 50°. Lines OA, OB and OC are drawn. The length of OB is 14.2 inches.

a Find to the *nearest inch* the length of the radius of the circle. [6]

b Using the value of the radius obtained in answer to *a* and using $\pi = \frac{22}{7}$, find to the *nearest square inch* the area of minor sector AOC. [4]

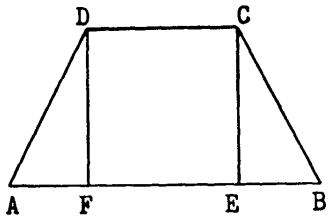
32 In the accompanying diagram, $ABCD$ is an isosceles trapezoid, $\angle B = 60°$, and DF and CE are altitudes.

a If EB is represented by x, represent CB in terms of x. [2]

b If DC exceeds CB by 2, represent the perimeter of the trapezoid in terms of x. [2]

c If the perimeter of the trapezoid is 44, find x. [2]

d Find the area of the trapezoid. [Answer may be left in radical form.] [4]



33 The four points A (1, 2), B (10, 5), C (9, 8) and D (0, 5) are the vertices of quadrilateral $ABCD$.

a Find the coordinates of the mid-point of AC; of BD. [2, 2]

b Show that $ABCD$ is a parallelogram. [1]

c Find the length of diagonal AC; of BD. [2, 2]

d Prove that parallelogram $ABCD$ is a rectangle. [1]

*34 *a* Find the slope of the line through A (1, —1) and B (5, 7). [3]

b Express in terms of K the slope of the line through C (0, 4) and D (3, K). [3]

c If AB is parallel to CD, find K. [2]

d Write the equation of line CD. [2]

* This question is based upon one of the optional topics in the syllabus.

Fill in the following lines:

Name of pupil.....Name of school.....

Part I

Answer all questions in part I. Each correct answer will receive 2 credits. No partial credit will be allowed.

1 Find the area of a circle whose radius is 6. [Answer may be left in terms of π.] 1.....

2 Find the length of an arc of 45° in a circle whose radius is 8. [Answer may be left in terms of π.] 2.....

3 The angles of a triangle are in the ratio 1 : 3 : 5. Find the number of degrees in the smallest angle of the triangle. 3.....

4 The perimeter of a regular polygon is 22 and its apothem, to the nearest integer, is 3. Find the area of the polygon. 4.....

5 From a point outside a circle, a tangent and a secant are drawn. If the tangent is 8 and the secant is 16, find the external segment of the secant. 5.....

6 Chord AB of a circle has an arc of 50°. Find the number of degrees in the acute angle formed by AB and the tangent to the circle at A. 6.....

7 Chords AB and CD intersect within a circle at point E. If arc $AC = 40°$ and arc $DB = 50°$, find the number of degrees in angle AEC. 7.....

8 Find the coordinates of the mid-point of the line segment whose end points are $(3, 6)$ and $(9, -2)$. 8.....

9 The coordinates of point P are $(8, 3)$. Find the distance of P from the origin. [Answer may be left in radical form.] 9.....

10 The line segment joining the mid-points of two sides of equilateral triangle ABC is 6. Find the perimeter of triangle ABC. 10.....

11 Find the area of a rhombus whose diagonals are 12 and 5. 11.....

12 In right triangle ABC, CD is the altitude to the hypotenuse. If $AD = 6$ and $AB = 10$, find AC. [Answer may be left in radical form.] 12.....

13 Find to the *nearest degree* the angle of elevation of the sun when a vertical 10-foot pole casts a shadow 4 feet long. 13.....

14 Find the area of an equilateral triangle whose side is 10. [Answer may be left in radical form.] 14.....

15 In triangle ABC, angle C is 60° and angle A is 65°. Name the shortest side of the triangle. 15.....

16 If corresponding sides of two similar polygons are in the ratio 1 : 3, find the ratio of their areas. 16.....

17 Write the equation of the locus of points equidistant from the points $(3, 6)$ and $(7, 6)$. 17.....

18 Is statement A the converse of statement B? [Answer *yes* or *no*.]
A Two triangles are similar if their corresponding angles are equal.
B The corresponding angles of two similar triangles are equal. 18.....

19 Can more than three angles of a pentagon be right angles? [Answer *yes* or *no*.] 19.....

[OVER]

Directions (20–24): Indicate the correct completion for *each* of the following by writing on the line at the right the letter *a, b* or *c*.

20 In triangle *ABC* side *AB* is twice median *CD*. Then triangle *ADC* and triangle *BDC* are always (*a*)congruent (*b*)equilateral (*c*)isosceles 20.....

21 The diagonals of a quadrilateral divide it into four congruent triangles if the quadrilateral is (*a*)a rectangle (*b*)a rhombus (*c*)an isosceles trapezoid 21.....

22 Given the statements:

(1) Two triangles are similar if three angles of one triangle are equal to three angles of the other.

(2) In a right triangle the square of the hypotenuse is equal to the sum of the squares of the other two sides.

(3) If in a right triangle the altitude is drawn to the hypotenuse, each leg is the mean proportional between the hypotenuse and the projection of that leg on the hypotenuse.

Which of the following represents the order in which the statements should be arranged so that they form a logical sequence?

(*a*) 3, 1, 2 (*b*) 1, 2, 3 (*c*) 1, 3, 2 22.....

23 A triangle is equal in area to a parallelogram. If the base of the triangle is equal to the base of the parallelogram, the altitude of the triangle is (*a*)equal to the altitude of the parallelogram (*b*)one half the altitude of the parallelogram (*c*)twice the altitude of the parallelogram 23.....

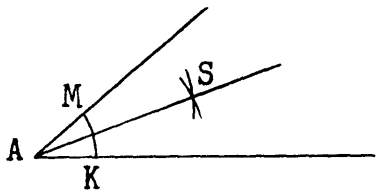
24 The accompanying diagram shows the construction of the bisector of angle *A*. An arc of a circle with its center at *A* is drawn cutting the sides of the angle at *M* and *K*, and *S* is located equidistant from *M* and *K*.

Which one of the following statements is used in proving that *AS* bisects angle *A*?

a An angle has one and only one bisector.

b Two triangles are congruent if two sides and the included angle of one are equal to the corresponding parts of the other.

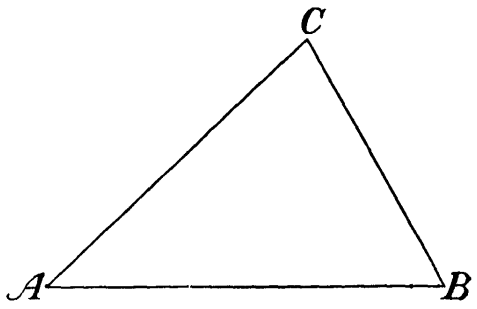
c Two triangles are congruent if the three sides of one are equal to the three sides of the other.



24.....

Directions (25): Leave all construction lines on your paper.

25 Find by construction the center of the circle that can be inscribed in triangle *ABC*.



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