The University of the State of New York

306TH HIGH SCHOOL EXAMINATION

TENTH YEAR MATHEMATICS

Monday, June 20, 1949 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II, III and IV (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in tenth year mathematics, (*c*) author of textbook used.

The minimum time requirement is four or five recitations a week for a school year.

Part II

Answer two questions from part II.

26 Prove that if two sides of a quadrilateral are equal and parallel, the figure is a parallelogram. [10]

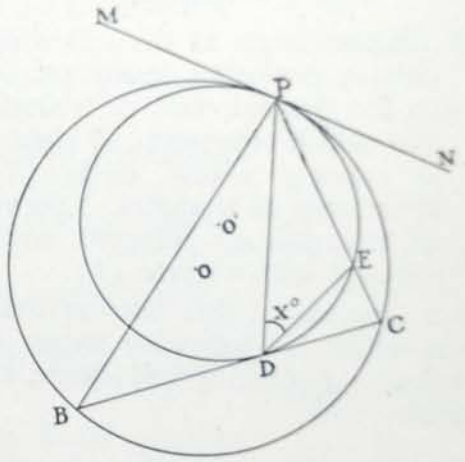
27 Secants PAB and PCD are drawn to a circle from the external point P and chords AB and CD thus formed are equal. Chord BD is drawn. Prove that

a arc AB = arc DC [1]

b $\angle B = \angle D$ [4]

c $AP = CP$ [5]

28 Two circles O and O' are tangent to line MN at point P as shown in the figure. Chord BC of the larger circle is tangent to the smaller circle at D, and chords PB, PD, PC and DE are drawn. Let the number of degrees in angle EDP be represented by x.



a Express in terms of x the number of degrees in arc PE; in angle EPN; in arc PC; in angle CBP. [4]

b Prove : $\angle PED = \angle PDB$ [3]

c Prove : $\triangle PED$ is similar to $\triangle PDB$ [3]

Part III

Answer two questions from part III.

29 *a* If $3x - 2$, $2x + 22$, and $2x - 8$ represent the number of degrees in the angles of a triangle, show that the triangle is isosceles. [4]

b Given the quadrilateral whose vertices are $A(2, 1)$, $B(6, 4)$, $C(8, 11)$, and $D(4, 8)$, find the coordinates of the mid-point of diagonal AC; of diagonal BD. Show that $ABCD$ is a parallelogram. [2, 2, 2]

30 Angle B of isosceles triangle ABC is 126° and base AC is 18 inches. Find, to the *nearest square inch*, the area of the triangle. [10]

31 The diagonals of a rhombus are in the ratio 5 : 12.

a If the shorter diagonal is represented by $5x$, represent the longer diagonal in terms of x. [1]

b Express the area of the rhombus in terms of x. [2]

c If the area of this rhombus is 120 square inches, find the length of each diagonal. [4]

d Find one side of the rhombus. [3]

[OVER]

32 *a* On a sheet of graph paper, draw two perpendicular axes and on this set of axes represent graphically the locus of points

(1) 5 units from the origin [2]

(2) 3 units from the x-axis [2]

b Write equations for the loci stated in part *a*. [2, 2]

c Find the coordinates of the points of intersection of the loci found in answer to part *a*. [2]

Part IV

Answer one question from part IV.

33 *a* Indicate the order in which the following statements should be arranged so that they form a *logical sequence*. Do not rewrite the statements. Merely indicate the sequence by using the number corresponding to the statement. [5]

(1) Through a given point only one straight line can be constructed parallel to a given line.

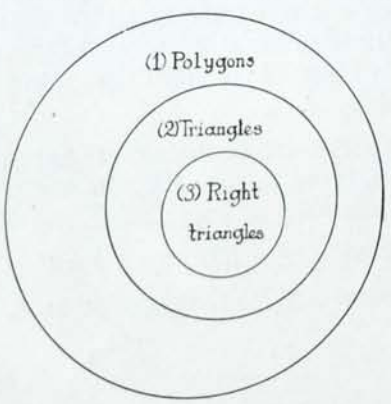
(2) The sum of the interior angles of a polygon of n sides is $(n-2)$ straight angles.

(3) Parallel lines are lines which lie in the same plane and do not intersect however far they are extended.

(4) The sum of the interior angles of a triangle is one straight angle.

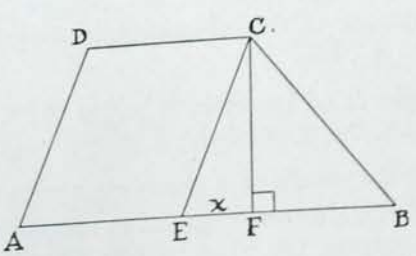
(5) When two parallel lines are cut by a transversal, either pair of alternate-interior angles are equal.

b A diagram such as the following is sometimes used to indicate the *sequence* in which certain geometric terms are defined. In this instance, since all right triangles belong to the general class of triangles, circle (3) which represents all right triangles is entirely within circle (2) which represents all triangles. Likewise, since all triangles are polygons, circle (2) is entirely within circle (1).



Make a diagram of this kind to indicate the sequence in which the following terms should be defined: polygon, parallelogram, square, rectangle, quadrilateral. [5]

34 The bases AB and CD of a trapezoid are 24 and 10 and the legs AD and BC are 13 and 15 respectively. CE is drawn parallel to DA and CF is perpendicular to AB as shown in the figure. Let EF be represented by x.



a Express FB in terms of x. [1]

b Using triangles EFC and BFC, write *two* expressions for $(CF)^2$ in terms of x. [2, 2]

c Find the value of x. [3]

d Find the area of the trapezoid. [2]

Fill in the following lines:

Name of pupil.....Name of school.....

Part I

Answer all questions in part I. Each correct answer will receive 2 credits. No partial credit will be allowed.

1 An exterior angle of a regular polygon is 40°. Find the number of sides of the polygon. 1.....

2 Two chords intersecting within a circle intercept opposite arcs of 40° and 60°. Find the number of degrees in the acute angle formed by the chords. 2.....

3 A secant and a tangent to a circle from an external point are 8 and 4 respectively. Find the external segment of the secant. 3.....

4 CD is the altitude on the hypotenuse of right triangle ABC. $AD = 2$ and $DB = 8$. Find CD. 4.....

5 A line parallel to side AB of triangle ABC intersects AC at D and BC at E. If $DC = 12$, $AD = 4$, and $EC = 18$, find BE. 5.....

6 Corresponding sides of two similar polygons are in the ratio 3:4. Find the ratio of their areas. 6.....

7 Find the diagonal of a square whose side is 8. [Answer may be left in radical form.] 7.....

8 Express the area of a regular polygon in terms of its apothem a and its perimeter p. 8.....

9 Find the altitude of an equilateral triangle whose side is 5. [Answer may be left in radical form.] 9.....

10 The angle of a sector of a circle is 60° and the radius of the circle is 6. Find the area of the sector. [Answer may be left in terms of π.] 10.....

11 A square is equal in area to a parallelogram whose base is 8 and whose altitude is 2. Find a side of the square. 11.....

12 A side of a triangle is 12. Find the length of the line segment joining the mid-points of the other two sides. 12.....

13 Find the area of a triangle whose vertices are $A(0, 0)$, $B(8, 0)$ and $C(2, 5)$. 13.....

14 The coordinates of point A are $(2, 7)$ and of point B are $(5, 3)$. Find the distance from A to B. 14.....

15 Write the equation of the straight line which is the locus of a point whose ordinate exceeds its abscissa by 2. 15.....

16 The sides of a rectangle are 8 and 10. Find, to the *nearest degree*, the angle formed by the diagonal and the longer side of the rectangle. 16.....

Directions (questions 17–23) — If the blank in each statement is replaced by one of the words *always, sometimes,* or *never,* the resulting statement is true. Select the word that will correctly complete *each* statement and write the word on the line at the right.

17 The diagonals AC and BD of quadrilateral $ABCD$ inscribed in a circle intersect at E. Triangle AED is ... similar to triangle BEC. 17.....

18 From external point A tangents AB and AC are drawn to a circle and chord BC is drawn. Triangle ABC is ... equilateral. 18.....

19 The locus of the center of a circle of given radius and tangent externally to a given circle is ... a circle. 19.....

[OVER]

20 The areas of two triangles having equal altitudes ... are to each other as their bases. 20.....

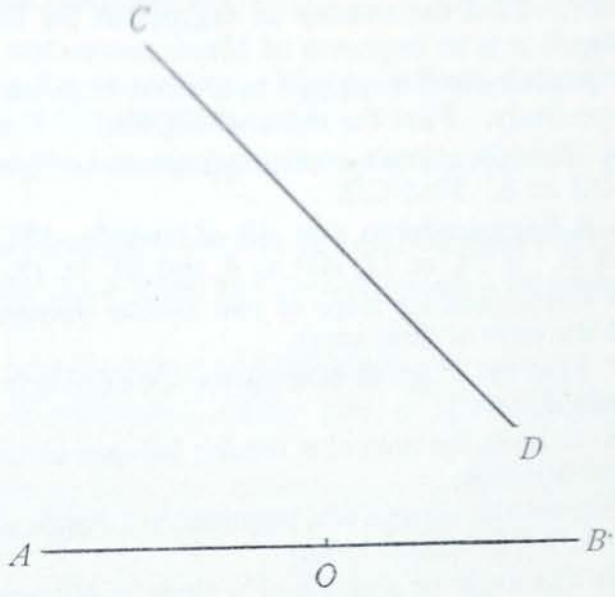
21 The medians of a triangle ... bisect each other. 21.....

22 If angle A of triangle ABC is greater than angle A' of triangle $A'B'C'$, then BC is ... greater than $B'C'$. 22.....

23 It is ... possible to construct triangle ABC if the given parts are side AB, angle A and the altitude on AB. 23.....

Directions (questions 24–25) — Leave all construction lines on your paper.

24 Construct a circle tangent to line AB at point O and having its center on line CD.



25 Divide line segment RS into *three* equal parts.

195